Day 112 **IGCSE 10-a-day: Model Solutions**

1 Point with a given gradient (quadratic)

A curve has equation $y = 2x^2 + 5x + 7$.
The gradient at the point P on the curve is 5.
Find the coordinates of P.

$\frac{dy}{dx} = 4x + 5$
$4x + 5 = 5$, so $4x = 0$
$x = 0$
$y = 7$
Answer: (0, 7)

2 Adding and scaling matrices

$A = \begin{pmatrix} 1 & 7 \\ 6 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} -6 & -3 \\ -5 & -3 \end{pmatrix}$.
(a) Work out A + B.
(b) Work out 4A − 3B.

(a) $A + B = \begin{pmatrix} -5 & 4 \\ 1 & -8 \end{pmatrix}$
(b) $4A = \begin{pmatrix} 4 & 28 \\ 24 & -20 \end{pmatrix}$ and $3B = \begin{pmatrix} -18 & -9 \\ -15 & -9 \end{pmatrix}$
$4A - 3B = \begin{pmatrix} 22 & 37 \\ 39 & -11 \end{pmatrix}$
Answer: (a) $\begin{pmatrix} \mathbf{-5} & \mathbf{4} \\ \mathbf{1} & \mathbf{-8} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{22} & \mathbf{37} \\ \mathbf{39} & \mathbf{-11} \end{pmatrix}$

3 Coordinates and gradient at the y-intercept

$y = 3x^3 + 2x^2 - 5x - 3$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = −3, so A is (0, −3)
$\frac{dy}{dx} = 9x^2 + 4x - 5$
When x = 0, $\frac{dy}{dx} = -5$
Answer: A = (0, −3), gradient −5

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 50 = 100°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 100 = 80°
Answer: (a) 100° (b) 80°

5 Fractions

Work out $1\frac{1}{4} \times 1\frac{1}{2} - 1\frac{1}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$

$\frac{15}{8} - \frac{4}{3} = \frac{13}{24}$

Answer: $\mathbf{\frac{13}{24}}$

6 Differentiating brackets with square roots

$y = \sqrt{x}(x + 1)$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x^{3/2} + x^{1/2}$

$x^{3/2} \rightarrow \frac{3}{2}x^{1/2}$, $x^{1/2} \rightarrow \frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}}$

7 Ratio and money transferred

Hassan and Oscar share some money in the ratio 6 : 5.
Hassan gives Oscar £14.
The ratio of their money is now 8 : 9.
How much money do they have altogether?

Let the amounts be 6k and 5k, so the total is 11k

Hassan now has $6k - 14$, which is $\frac{8}{17}$ of the total

$17(6k - 14) = 8 \times 11k$
$102k - 238 = 88k$, so $14k = 238$ and $k = 17$
Total = 11×17 = £187
Answer: £187

8 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 51%.
Work out the value of x.

Yearly multiplier = $1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.49$

Take the square root: $1 - \frac{x}{100} = 0.7$

$\frac{x}{100} = 0.3$, so $x = 30$

Answer: x = 30

9 Equations with algebraic fractions

Solve $\frac{6}{x-1} + \frac{2}{x-4} = 1$

Multiply every term by $(x - 1)(x - 4)$: $6(x - 4) + 2(x - 1) = (x - 1)(x - 4)$

Expand and rearrange so that one side is 0: $x^2 - 13x + 30 = 0$

Factorise: $(x - 10)(x - 3) = 0$

$x = 3$ or $x = 10$

Answer: x = 3 or x = 10

10 Bounds: finding an unknown length

A rectangle has area 36 cm^2, correct to 2 significant figures.

Its length is 5.900000000000000355271367880050092935562133789062 5 cm, correct to 2 significant figures.

Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $35.5 \leq A < 36.5$. Length: $5.850000000000000355271367880050092935562133789062 5 \leq l < 5.950000000000000355271367880050092935562133789062 5$

UB width = UB area ÷ LB length = 36.5 ÷ 5.850000000000000355271367880050092935562133789062 5 = 6.24

Answer: 6.24 cm