Day 111 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 10:55 (London time).
It takes 9 hours 30 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 10:55 + 9 h 30 min = 20:25
Tokyo is 8 hours ahead of London, so add 8 hours
04:25 the next day
Answer: 04:25 the next day

2 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 120 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mark greater than 55.

(a) Median at cumulative frequency 60, so median = 30
Lower quartile at 30, so LQ = 20. Upper quartile at 90, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 55, cumulative frequency = 106
Number greater = 120 − 106 = 14
Answer: (a) median 30, IQR 20 (b) 14

3 Equations with fractions

Solve $\frac{x+1}{2} + \frac{x-6}{8} = -4$

Multiply every term by 8: $4(x+1) + (x-6) = -32$
Expand: $5x - 2 = -32$
$5x = -30$
$x = -6$
Answer: $x = -6$

4 Arc length and sector area

A sector of a circle has radius 13 cm and angle 45°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{45}{360}$

Arc length = $\frac{45}{360} \times 2 \times \pi \times 13 = \frac{13}{4}\pi$ cm

(b) Area = $\frac{45}{360} \times \pi \times 13^2 = \frac{169}{8}\pi$ cm^2

Answer: (a) $\frac{13}{4}\pi$ cm (b) $\frac{169}{8}\pi$ cm^2

5 Solving trigonometric equations

Solve $\cos 3x = 0.8$ for $0 \le x \le 120$.
Give your answers correct to 1 decimal place where necessary. (Calculator)

Let $u = 3x$, so $\cos u = 0.8$ with $0 \le u \le 360$
Principal value $u = 36.9°$
All values of u: 36.9, 323.1
Divide by 3: x = 12.3, 107.7
Answer: x = 12.3, 107.7

6 Intersection and gradient

The curve $y = 2x^2 - 5x + 5$ and the line $y = -3x + 5$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$2x^2 - 5x + 5 = -3x + 5$ gives $2x^2 - 2x = 0$, so $2x(x - 1) = 0$
$x = 0$ or $x = 1$, giving A (0, 5) and B (1, 2)
$\frac{dy}{dx} = 4x - 5$: at A it is -5, at B it is -1
Answer: (a) A (0, 5), B (1, 2) (b) -5 at A, -1 at B

7 Functions that commute

$f(x) = 5x - 4$ and $g(x) = kx - 3$, where k is a constant.
$fg(x) = gf(x)$ for all values of x.
Find the value of k.

$fg(x) = 5(kx - 3) - 4 = 5kx - 19$
$gf(x) = k(5x - 4) - 3 = 5kx - 4k - 3$
Equate the constant terms: $-19 = -4k - 3$
$-4k = -16$, so $k = 4$
Answer: k = 4

8 Open box from a square sheet

Squares of side x cm are cut from the corners of a 18 cm square cardboard sheet, which is folded into an open box of volume V cm^3.
(a) Show that $V = 4x^3 - 72x^2 + 324x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side $18 - 2x$, so $V = x(18 - 2x)^2 = x(324 - 72x + 4x^2) = 4x^3 - 72x^2 + 324x$
$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$, so $x^2 - 12x + 27 = 0$
$12(x - 3)(x - 9) = 0$, so $x = 3$ or $x = 9$
$x = 9$ is rejected (9 is more than half of 18). $\frac{d^2V}{dx^2} = 24x - 144$, which is -72 when $x = 3$, so a maximum
$V = 432$
Answer: x = 3, V = 432 cm^3

9 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles. (Calculator)
An athlete runs 5 complete laps around the track.
Work out the total distance run, to the nearest 10 metres.
The diagram is not drawn accurately.

Boundary of one lap $= 2 \times 60 + 2 \times \pi \times 20 = 245.66$ m
5 laps $= 5 \times 245.66 = 1228.3$ m
= 1230 m to the nearest 10 m
Answer: 1230 m

10 Normal meets the curve again (quadratic)

The normal to the curve $y = -x^2 + x - 4$ at the point P where $x = 0$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = -2x + 1$: at P, gradient = 1 and $y = -4$
Normal: $y = -x - 4$
Intersection: $-x^2 + 2x = 0$
One root is $x = 0$, and the sum of the roots is 2
So $x_Q = 2 - 0 = 2$
$y = -6$
Answer: (2, −6)