Day 110 **IGCSE 10-a-day: Model Solutions**

1 Turning point of a quadratic by differentiation

$f(x) = -2x^2 - 8x + 9$

Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = -4x - 8$

$f'(x) = 0$: $-4x - 8 = 0$, so $x = -2$

$f(-2) = 17$

$f''(x) = -4$, which is negative, so it is a maximum

Answer: $(-2, 17)$, maximum

2 Inequalities

(a) Solve $1 - 2x < -3$

(b) n is an integer. Write down all the values of n that satisfy

$-17 \leq 4n - 1 \leq 3$

(a) Subtract 1: $-2x < -4$

Divide by -2 and reverse the inequality sign: $x > 2$

(b) Add 1: $-16 \leq 4n \leq 4$

Divide by 4: $-4 \leq n \leq 1$

Integers: $-4, -3, -2, -1, 0, 1$

Answer: (a) $x > 2$ (b) $-4, -3, -2, -1, 0, 1$

3 Points with a given gradient (cubic)

The curve $y = x^3 - 3x^2 - 16x + 7$ has gradient -7 at two points.

Find the coordinates of these points.

$\frac{dy}{dx} = 3x^2 - 6x - 16$

$3x^2 - 6x - 16 = -7$, so $3x^2 - 6x - 9 = 0$

Divide by 3: $x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$, so $x = -1$ or $x = 3$

Substitute into y: $(-1, 19)$ and $(3, -41)$

Answer: $(-1, 19)$ and $(3, -41)$

4 Position vector of a point dividing a line

The position vectors of A and P are $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$.

P lies on AB so that AP : PB = 1 : 2.

Find the position vector of B.

$\mathbf{AP} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} - \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

AP is 1 part, so one part is $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

AB is 3 parts: $\mathbf{AB} = 3 \times \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \begin{pmatrix} -14 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-14} \\ \mathbf{2} \end{pmatrix}$

5 Exact trigonometric values

Work out the exact value of sin 60° × sin 45°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\sin 60° \times \sin 45° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4}$

Answer: $\mathbf{\frac{\sqrt{6}}{4}}$

6 Powers of standard form

Work out the square root of 4.84×10^{10}.
Give your answer in standard form.

$4.84 = 2.2 \times 2.2$
$\sqrt{(10^{10})} = 10^5$
$\sqrt{(4.84 \times 10^{10})} = 2.2 \times 10^5$
Answer: $\mathbf{2.2 \times 10^5}$

7 Rates of work

Tap A can fill an empty tank in 10 minutes.
Tap B can fill an empty tank in 40 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{10}$ and Tap B does $\frac{1}{40}$

Together: $\frac{1}{10} + \frac{1}{40} = \frac{50}{400}$

Time = $\frac{400}{50}$ = 8 minutes

Answer: 8 minutes

8 Increasing for all x

$f(x) = x^3 + kx^2 + 75x - 5$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 75$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 75 < 0$, so $4k^2 < 900$
$k^2 < 225$, so $-15 < k < 15$
Answer: $\mathbf{-15 < k < 15}$

9 Exact trigonometric ratios from a given ratio

x is an acute angle and $\tan x = \frac{3}{7}$.

(a) Find the exact value of sin x.
(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 3 and adjacent side 7
Hypotenuse = $\sqrt{3^2 + 7^2} = \sqrt{58}$
(a) $\sin x = \frac{3}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$
(b) $\cos x = \frac{7}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$
Answer: (a) $\mathbf{\frac{3\sqrt{58}}{58}}$ **(b)** $\mathbf{\frac{7\sqrt{58}}{58}}$

10 Three picks without replacement

A bag has 4 black and 7 green counters.
Three counters are taken at random without replacement.
(a) Work out the probability that all three are green.
(b) Work out the probability that at least one is black.

(a) $\frac{7}{11} \times \frac{6}{10} \times \frac{5}{9} = \frac{7}{33}$

(b) At least one black is the opposite of all three green

$1 - \frac{7}{33} = \frac{26}{33}$

Answer: (a) $\frac{7}{33}$ (b) $\frac{26}{33}$