Day 11 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = 4x^4 + x - \frac{2}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = 4x^4 + x - 2x^{-2}$

$4x^4 \to 16x^3$, $x \to 1$, $-2x^{-2} \to 4x^{-3}$

$\frac{dy}{dx} = 16x^3 + 1 + 4x^{-3}$

Answer: $\mathbf{\frac{dy}{dx} = 16x^3 + 1 + \frac{4}{x^3}}$

2 Range of a quadratic function

$f(x) = -2x^2 + 4x - 5$ for $x \geq 1$.
Find the range of f.

Complete the square: $f(x) = -2(x - 1)^2 - 3$

$x \geq 1$ includes the turning point, so the maximum value is -3

Answer: $f(x) \leq -3$

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 67°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 67 = 134°$

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = $360 - 90 - 90 - 134 = 46°$

Answer: (a) 134° (b) 46°

4 Rationalising the denominator

Rationalise the denominator of $\frac{6}{4\sqrt{5}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{5}$:

$\frac{6}{4\sqrt{5}} = \frac{6\sqrt{5}}{20}$

$= \frac{3\sqrt{5}}{10}$

Answer: $\mathbf{\frac{3\sqrt{5}}{10}}$

5 Turning points of a cubic

The diagram shows a sketch of the curve $y = 2x^3 - 6x^2 - 18x$.
The curve has turning points at A and B.
Find the coordinates of A and B and state whether each is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 - 12x - 18 = 6(x + 1)(x - 3)$

$\frac{dy}{dx} = 0$ gives $x = -1$ or $x = 3$

$y = 10$ when $x = -1$ and $y = -54$ when $x = 3$

$\frac{d^2y}{dx^2} = 12x - 12$: at $x = -1$ it is -24 (maximum), at $x = 3$ it is 24 (minimum)

Answer: A $(-1, 10)$ maximum, B $(3, -54)$ minimum

6 Matrix of a transformation

Write down the matrix that represents each transformation.
(a) Stretch, scale factor 2, x-axis invariant
(b) Enlargement, scale factor -1, centre the origin

(a) Find where (1, 0) and (0, 1) go: they are the columns of the matrix

$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$

7 Algebraic proof (harder)

Prove that the difference between the cubes of any two consecutive integers is always an odd number.

Let the integers be k and k + 1.

$(k + 1)^3 - k^3 = k^3 + 3k^2 + 3k + 1 - k^3 = 3k^2 + 3k + 1$

$= 3k(k + 1) + 1$

k(k + 1) is even, so 3k(k + 1) is even, and adding 1 gives an odd number

Answer: Proved

8 Stationary point with a 1/x term

A curve has equation $y = x^2 + \frac{2}{x}$.

(a) Show that the curve has a stationary point at x = 1.
(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 2x - \frac{2}{x^2}$

When x = 1: $\frac{dy}{dx} = 2 - 2 = 0$

y = 3

$\frac{d^2y}{dx^2} = 2 + \frac{4}{x^3}$

At x = 1: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: y = 3, minimum

9 Cone made from a sector

A sector of a circle has radius 16 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 16 = 16\pi$ cm

This is the circumference of the base: $2\pi r = 16\pi$, so r = 8 cm

(b) Slant height = 16 cm, so height$^2 = 16^2 - 8^2 = 192$, height = $8\sqrt{3}$ cm

Volume = $\frac{1}{3} \times \pi \times 8^2 \times 8\sqrt{3} = \frac{512\sqrt{3}}{3}\pi$ cm^3

Answer: (a) 8 cm (b) $\mathbf{\frac{512\sqrt{3}}{3}\pi}$ **cm**^{3}

10 Winning a match

Oscar and Jamal play a match. The first to win 2 table tennis games wins the match.

The probability that Oscar wins any one of the table tennis games is $\frac{1}{3}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Oscar wins then loses) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

P(Oscar loses then wins) = $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Total = $2 \times \frac{2}{9} = \frac{4}{9}$

Answer: $\frac{4}{9}$