Day 109 **IGCSE 10-a-day: Model Solutions**

1 Percentage change

The price of a phone is increased by 25%.
The new price is then decreased by 25%.
Work out the overall percentage change in the price.

Multipliers: 1.25 and 0.75
Overall multiplier = 1.25 × 0.75 = 0.9375
0.9375 − 1 = −0.0625, so an overall decrease of 6.25%
Answer: 6.25% decrease

2 Mean of a changing set

The mean of 5 numbers is 99.
A new number is added and the mean of all 6 numbers is 86.
Work out the new number.

Total of the 5 numbers = 5 × 99 = 495
New total = 6 × 86 = 516
New number = 516 − 495 = 21
Answer: 21

3 Turning point of a quadratic by differentiation

$f(x) = -x^2 + 6x + 8$
Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = -2x + 6$
$f'(x) = 0$: $-2x + 6 = 0$, so $x = 3$
$f(3) = 17$
$f''(x) = -2$, which is negative, so it is a maximum
Answer: (3, 17), maximum

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 20: circumference = 2 × π × 20 = 40π
The two straight sides: 2 × 80 = 160
Boundary = (160 + 40π) m
(b) The rectangle is 80 m by 2 × 20 = 40 m, so its area is 80 × 40 = 3200. Circle: π × 20^2 = 400π
Area = (3200 + 400π) m^2
Answer: (a) (160 + 40π) m (b) (3200 + 400π) m^2

5 Fractions of fractions

Beth spent $\frac{2}{3}$ of her money on a coat.

She then spent $\frac{1}{7}$ of the money that was left on a bag.

She then had £720 left.
How much money did Beth have to start with?

After the first part, $\frac{1}{3}$ remains

After the second, $\frac{6}{7}$ of $\frac{1}{3}$ remains = $\frac{2}{7}$ of the total

$\frac{2}{7}$ of the total is £720

Total = £720 ÷ $\frac{2}{7}$ = £2,520

Answer: £2,520

6 Inverse of a rational function

$f(x) = \frac{-5x - 3}{4x + 5}$, $x \neq -\frac{5}{4}$.

Find $f^{-1}(x)$.

$y = \frac{-5x - 3}{4x + 5}$, so $y(4x + 5) = -5x - 3$

$4xy + 5y = -5x - 3$

$4xy + 5x = -3 - 5y$

$x(4y + 5) = -3 - 5y$, so $x = \frac{-3 - 5y}{4y + 5}$

$f^{-1}(x) = \frac{-3 - 5x}{4x + 5}$

Answer: $f^{-1}(x) = \frac{-3 - 5x}{4x + 5}$

7 Solve f′(x) = 0 with a reciprocal term

$f(x) = 2x^3 + \frac{96}{x}$

Find the values of x for which $f'(x) = 0$.

$f'(x) = 6x^2 - \frac{96}{x^2}$

$f'(x) = 0$: $6x^2 = \frac{96}{x^2}$

$x^4 = 16$

$x = 2$ or $x = -2$

Answer: $x = 2$ or $x = -2$

8 Transformed curve from a diagram

The diagram shows the graph of $y = f(x)$ with a minimum point at (3, 3).
The dashed curve C is the image of $y = f(x)$ after a single transformation.
The maximum point of C is (3, −3).
Write down the equation of C in terms of f.

The minimum point (3, 3) has moved to (3, −3)
The x-coordinate is unchanged and the y-coordinate has changed sign
This is a reflection in the x-axis

Answer: $y = -f(x)$

9 Composite functions: fg and gf

$f(x) = -x - 2$ and $g(x) = x^2 - 6$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 6) = -x^2 + 4$

$gf(x) = g(-x - 2) = (-x - 2)^2 - 6 = x^2 + 4x - 2$

Set them equal and rearrange: $2x^2 + 4x - 6 = 0$

Divide by 2 and factorise: $(x - 1)(x + 3) = 0$

$x = -3$ or $x = 1$

Answer: $x = -3$ or $x = 1$

10 Rectangle under a parabola

The diagram shows the curve $y = 24 - 2x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).

(a) Show that the area of ABCD is $-4x^3 + 48x$.

(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $24 - 2x^2$, so area = $2x(24 - 2x^2) = -4x^3 + 48x$

$\frac{dA}{dx} = -12x^2 + 48 = 0$, so $x^2 = 4$ and $x = 2$ ($x > 0$)

$\frac{d^2A}{dx^2} = -24x < 0$ when $x = 2$, so a maximum

Area = $48 \times 2 - 4 \times 2^3 = 64$

Answer: 64