Day 108 **IGCSE 10-a-day: Model Solutions**

1 Square roots as powers

$y = -6x^{5/2} + 3x^2 - \frac{6}{\sqrt{x}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -6x^{5/2} + 3x^2 - 6x^{-1/2}$

$-6x^{5/2} \rightarrow -15x^{3/2}$, $3x^2 \rightarrow 6x$, $-6x^{-1/2} \rightarrow 3x^{-3/2}$

$\frac{dy}{dx} = -15x^{3/2} + 6x + 3x^{-3/2}$

Answer: $\mathbf{\frac{dy}{dx} = -15x^{3/2} + 6x + 3x^{-3/2}}$

2 Adding and scaling matrices

$A = \begin{pmatrix} 7 & 2 \\ -2 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$.

Find the matrix X such that A + X = B.

$X = B - A$

$X = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 7 & 2 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} -5 & 1 \\ 2 & -6 \end{pmatrix}$

Answer: $\mathbf{X = \begin{pmatrix} -5 & 1 \\ 2 & -6 \end{pmatrix}}$

3 Solving quadratics by factorising

Solve $9x^2 + 10 = 21x$

Rearrange so that one side is 0: $9x^2 - 21x + 10 = 0$

Factorise: $(3x - 2)(3x - 5) = 0$

$3x - 2 = 0$ or $3x - 5 = 0$

$x = \frac{2}{3}$ or $x = \frac{5}{3}$

Answer: $\mathbf{x = \frac{2}{3}}$ **or** $\mathbf{x = \frac{5}{3}}$

4 Pressure

A concrete slab in the shape of a cuboid weighs 510 N.

Its smallest face has area 100 cm^2. Its largest face has area 500 cm^2.

Pressure = force ÷ area

Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 500 cm^2 = 0.05 m^2

(a) Greatest pressure is on the smallest face: 510 ÷ 0.01 = 51,000 N/m^2

(b) Least pressure is on the largest face: 510 ÷ 0.05 = 10,200 N/m^2

Answer: (a) 51,000 N/m^2 (b) 10,200 N/m^2

5 Percentage change

A 30% increase followed by a 25% decrease is equivalent to a single percentage change.

Work out the single percentage change that has the same effect.

Multipliers: 1.3 and 0.75

Overall multiplier = 1.3 × 0.75 = 0.975

0.975 − 1 = −0.025, so an overall decrease of 2.5%

Answer: 2.5% decrease

6 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 7)$ cm, $(x - 8)$ cm and $(x - 6)$ cm.
The longest side is $(x - 6)$ cm.
(a) Show that $x^2 - 18x + 77 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 7)^2 + (x - 8)^2 = (x - 6)^2$
Expand: $x^2 - 14x + 49 + x^2 - 16x + 64 = x^2 - 12x + 36$
Rearrange: $x^2 - 18x + 77 = 0$
(b) Factorise: $(x - 11)(x - 7) = 0$, so $x = 11$ (the other root, $x = 7$, does not give valid side lengths)
The shorter sides are 4 cm and 3 cm, so area $= \frac{1}{2} \times 4 \times 3 = 6$ cm^2

Answer: (b) 6 cm^2

7 Area of a triangle: ½ab sin C

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures. (Calculator)

Area = ½ × a × b × sin C
= ½ × 10 × 10.4 × sin 47°
= 38.0

Answer: 38.0 cm^2

8 Open box of fixed surface area

An open box has a square base of side x cm and height h cm. The total outside surface area is 108 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(108x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 108$, so $h = \frac{108 - x^2}{4x}$

$V = x^2h = \frac{x(108 - x^2)}{4} = \frac{1}{4}(108x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 27 = 0$, so $x^2 = 36$ and $x = 6$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -9 < 0$, so a maximum

$V = 108$

Answer: 108 cm^3

9 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 29^2 - 20^2 = 441$
AM = 21 cm
AB = 2 × 21 = 42 cm
(b) sin(angle AOM) $= \frac{21}{29}$
Angle AOM = 46.40°, so angle AOB = 2 × 46.40 = 92.8°

Answer: (a) 42 cm (b) 92.8°

10 Nature from the gradient either side

$y = -2x^3 + 6x^2 + 18x - 4$

(a) Show that there is a stationary point at $x = 3$.

(b) By finding the gradient when $x = \frac{5}{2}$ and when $x = \frac{7}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 12x + 18$, so when $x = 3$: $\frac{dy}{dx} = 0$

$x = \frac{5}{2}$: gradient = $\frac{21}{2}$ (positive)

$x = \frac{7}{2}$: gradient = $-\frac{27}{2}$ (negative)

The gradient goes from positive to 0 to negative, so (3, 50) is a maximum

Answer: (b) maximum