Day 107 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

$y = -3x^4 - 8x^3 - 6x - 2$

Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-3x^4 \to -12x^3$, $-8x^3 \to -24x^2$, $-6x \to -6$

$\frac{dy}{dx} = -12x^3 - 24x^2 - 6$

Answer: $\frac{dy}{dx} = -12x^3 - 24x^2 - 6$

2 LCM problems

A red light flashes every 12 minutes.
A blue light flashes every 15 minutes.
A green light flashes every 20 minutes.
They all flash together at 6:00 am.
(a) At what time do they next flash together?

Find the LCM of 12, 15 and 20

$12 = 2^2 \times 3$, $15 = 3 \times 5$, $20 = 2^2 \times 5$

LCM $= 2^2 \times 3 \times 5 = 60$

60 minutes after 6:00 am is 7:00 am

Answer: 7:00 am

3 Inverse and repeated functions

$f(x) = 3x + 3$.

(a) Find $f^{-1}(-3)$.
(b) Express $ff(x)$ in the form $px + q$.

(a) Solve $3x + 3 = -3$: $3x = -6$

$x = -2$ so $f^{-1}(-3) = -2$

(b) $ff(x) = 3(3x + 3) + 3 = 9x + 12$

Answer: (a) -2 (b) $ff(x) = 9x + 12$

4 Percentages

R is 50% less than Q.
Q is 12% more than B.
Work out R as a percentage of B.

$R = 0.5 \times Q$ and $Q = 1.12 \times B$

$R = 0.5 \times 1.12 \times B = 0.56 \times B$

$0.56 \times 100 = 56\%$

Answer: 56%

5 Enlargement

Triangle P has vertices A(1, 4), B(3, 2) and C(−2, 4).
Triangle Q has vertices A'(3, 7), B'(7, 3) and C'(−3, 7).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 2, x-step from A' to B' = 4

Scale factor = 4 ÷ 2 = 2

The centre C satisfies: $C + 2(A - C) = A'$, so $(1 - 2)C = A' - 2A$

x: $(-1) \times x = 3 - 2 \times 1 = 1$, so $x = -1$

y: $(-1) \times y = 7 - 2 \times 4 = -1$, so $y = 1$

Answer: Enlargement, scale factor 2, centre (−1, 1)

6 Venn diagram with three sets: algebra

The Venn diagram shows information about 45 shoppers.
A: bought fruit B: bought bread C: bought milk
(a) Find the value of x.
(b) Find n(A) and n(B ∪ C)′.

(a) $4x + 21 = 45$
$4x = 24$, so $x = 6$
(b) Substitute $x = 6$ to fill in every region
$n(A) = 1 + 3 + 6 + 8 = 18$
$n(B \cup C)' = 8 + 3 = 11$ (A only plus outside)
Answer: (a) x = 6 (b) n(A) = 18, n(B ∪ C)′ = 11

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 4n - 6$.
(a) Work out the 12th term.
(b) Is 249 a term of the sequence? You must show your working.

(a) $12^2 + 4 \times 12 - 6 = 186$
(b) Solve $n^2 + 4n - 6 = 249$, so $n^2 + 4n - 255 = 0$
$b^2 - 4ac = 4^2 - 4 \times 1 \times (-255) = 1036$, which is not a square number
So n is not a whole number, and 249 is not a term
Answer: (a) 186 (b) No

8 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by
$v = 6t - t^2$ for $0 \leq t \leq 6$
Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -2t + 6$
Greatest speed when $a = 0$: $-2t + 6 = 0$, so $t = 3$
$\frac{d^2v}{dt^2} = -2 < 0$, so this is a maximum
$v = 6 \times 3 - 1 \times 3^2 = 9$
Answer: t = 3 s, greatest speed 9 m/s

9 Finding the rate of interest

Liam invested £10,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £11,236.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $10000 \times m^2 = 11236.00$
$m^2 = 11236.00 \div 10000 = 1.1236$
$m = \sqrt{1.1236} = 1.06$
Rate $= (1.06 - 1) \times 100 = 6\%$
Answer: 6% per year

10 Where two tangents meet

The curve C has equation $y = -x^2 - 4x - 3$.
The points A and B on C have x coordinates 0 and 3.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = -2x - 4$. At A: gradient -4, A = $(0, -3)$

Tangent at A: $y = -4x - 3$. At B: gradient -10, B = $(3, -24)$

Tangent at B: $y = -10x + 6$

Equate: $-4x - 3 = -10x + 6$, so $x = \frac{3}{2}$ and $y = -9$

Answer: T $(\frac{3}{2}, -9)$