Day 106 **IGCSE 10-a-day: Model Solutions**

1 Independent events and probability rules

A and B are two events.
P(A) = 0.8, P(B) = 0.4 and P(A or B) = 0.85
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.85 = 0.8 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.35$
(b) $P(A) \times P(B) = 0.8 \times 0.4 = 0.32$
This is not equal to 0.35, so A and B are not independent
Answer: (a) 0.35 (b) No

2 Comparing costs

Sparkle Cleaners has a call-out fee of £4 plus £5.50 per hour.
Fresh Start has a call-out fee of £25 plus £2 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 11 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $4 + 5.50n = 25 + 2n$
$3.50n = 21$, so $n = 6$
(b) Sparkle Cleaners: £4 + £5.50 × 11 = £64.50
Fresh Start: £25 + £2 × 11 = £47
Fresh Start is cheaper by £17.50
Answer: (a) 6 (b) Fresh Start, by £17.50

3 Solve a linear inequality

Solve $-6x + 1 \leq -4x + 19$

Collect x terms: $-2x \leq 18$
Divide by -2 and reverse the inequality sign: $x \geq -9$
Answer: $x \geq -9$

4 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{32}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 4.

(b) Find the value of t when the temperature is falling at 32 °C per minute.

$T = 20 + 32t^{-1}$, so $\frac{dT}{dt} = -32t^{-2}$

When t = 4: $\frac{dT}{dt} = -2$

$-\frac{32}{t^2} = -32$, so $t^2 = 1$ and $t = 1$

Answer: (a) -2 °C/min (b) t = 1

5 Exact lengths and areas

A right-angled triangle has base $(4 + \sqrt{7})$ cm and height $(4 - \sqrt{7})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area $= \frac{1}{2} \times (4 + \sqrt{7})(4 - \sqrt{7})$

$(4 + \sqrt{7})(4 - \sqrt{7}) = 16 - 7 = 9$

Area $= \frac{1}{2} \times 9 = \frac{9}{2}$

Answer: $\frac{9}{2}$ cm^2

6 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x - 2)$ cm, $(x + 5)$ cm and $(x + 7)$ cm. The longest side is $(x + 7)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x - 2)^2 + (x + 5)^2 = (x + 7)^2$
Expand and collect terms: $x^2 - 8x - 20 = 0$
Factorise: $(x - 10)(x + 2) = 0$
$x = 10$ or $x = -2$, and x must be positive, so $x = 10$
(b) The two shorter sides are 8 cm and 15 cm

Area $= \frac{1}{2} \times 8 \times 15 = 60$ cm^2

Answer: (a) x = 10 (b) 60 cm^2

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx - 5$.
The curve has a turning point at M(1, −4).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when $x = 1$, so $b = -2a$
The point (1, −4) is on the curve: $1a + 1b - 5 = -4$
Substitute $b = -2a$: $-1a = 1$, so $a = -1$ and $b = 2$
Answer: a = −1, b = 2

8 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £11.00 per kg and type B costs £14.00 per kg.
They are mixed in the ratio A : B = 1 : 3.
The mixture is sold for £18.55 per kg.
Work out the percentage profit.

Cost of 4 kg of mixture = 1 × £11.00 + 3 × £14.00 = £53.00
Cost per kg = £53.00 ÷ 4 = £13.25
Profit per kg = £18.55 − £13.25 = £5.30
Percentage profit = 5.30 ÷ 13.25 × 100 = 40%
Answer: 40%

9 Cylinder of fixed surface area

A closed cylinder (a can) has radius r cm, height h cm and total surface area 24π cm^2.
(a) Show that its volume, V cm^3, is $V = 12\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that $h = 2r$ when V is a maximum.

$2\pi r^2 + 2\pi rh = 24\pi$, so $h = \frac{12 - r^2}{r}$
$V = \pi r^2 h = \pi r(12 - r^2) = 12\pi r - \pi r^3$
$\frac{dV}{dr} = 12\pi - 3\pi r^2 = 0$, so $r^2 = 4$ and $r = 2$
$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{12 - 4}{2} = 4 = 2r$
$V = \pi \times 2^2 \times 4 = 16\pi$
Answer: 16π cm^3

10 Triangle shapes from position vectors

A = (−4, −2), B = (−9, 2) and C = (1, 2).

Show that triangle ABC is isosceles.

$\mathbf{AB} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$|AB|^2 = 25 + 16 = 41$ and $|AC|^2 = 25 + 16 = 41$

$|AB| = |AC| = \sqrt{41}$, so the triangle is isosceles

Answer: $|\mathbf{AB}| = |\mathbf{AC}|$