Day 105 **IGCSE 10-a-day: Model Solutions**

1 Gradient function from a given point

A curve has equation $y = -2x^3 + 12x^2 - 18x + 9$.
The point A (0, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute x = 0 into the equation of the curve
y = 9 when x = 0, so k = 9
$\frac{dy}{dx} = -6x^2 + 24x - 18$
At A: gradient = −18
Answer: k = 9, gradient = −18

2 Angles in polygons

Each interior angle of a regular polygon is 171°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 171° = 9°
Number of sides = 360° ÷ 9° = 40
Answer: 40

3 Expand then differentiate

$y = (x - 4)(x - 3)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 - 7x + 12$
$x^2 \rightarrow 2x$, $-7x \rightarrow -7$
$\frac{dy}{dx} = 2x - 7$
Answer: $\mathbf{\frac{dy}{dx} = 2x - 7}$

4 Average speed

A car travels the first half of a journey at 20 km/h.
It travels the second half of the same journey at 80 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 80 km (a common multiple of 20 and 80)
Time for first half = 80 ÷ 20 = 4 hours
Time for second half = 80 ÷ 80 = 1 hours
Average speed = total distance ÷ total time = 160 ÷ 5 = 32 km/h
Answer: 32 km/h

5 Reverse percentages

The price of a camera is decreased by 30%.
The new price is then decreased by 10%.
The final price is £516.60.
Work out the original price of the camera.

Multipliers: 0.7, 0.9
Overall multiplier = 0.7 × 0.9 = 0.63
Original = final ÷ multiplier = £516.60 ÷ 0.63
= £820
Answer: £820

6 Show that lines are parallel

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
M is on OA and N is on OB, with OM : MA = ON : NB = 2 : 3.
(a) Show that MN is parallel to AB.
(b) Write down the ratio MN : AB.

(a) $\mathbf{OM} = \frac{2}{5}\mathbf{a}$ and $\mathbf{ON} = \frac{2}{5}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\frac{2}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$\mathbf{b} - \mathbf{a} = \mathbf{AB}$, so $\mathbf{MN} = \frac{2}{5}\mathbf{AB}$: a multiple of $\mathbf{AB}$, so MN is parallel to AB

(b) MN : AB = 2 : 5

Answer: (b) 2 : 5

7 Iteration: rearranging equations

(a) Show that the equation $x^3 + 7x - 30 = 0$ can be rearranged to give $x = \sqrt[3]{30 - 7x}$ (Calculator)
(b) Use the iteration formula $x_{n+1} = \sqrt[3]{30 - 7x_n}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 + 7x - 30 = 0$: $x^3 = 30 - 7x$

Take the cube root of both sides: $x = \sqrt[3]{30 - 7x}$

(b) $x_1 = \sqrt[3]{30 - 7 \times 2} = 2.51984...$

$x_2 = \sqrt[3]{30 - 7 \times x_1} = 2.31217...$

$x_1 = 2.520$ and $x_2 = 2.312$ (3 d.p.)

Answer: (b) $x_1 = 2.520$, $x_2 = 2.312$

8 Tangent meets the curve again

A curve has equation $y = x^3 - 4x^2 + 7x + 6$.
The tangent to the curve at the point A, where x = 2, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 8x + 7$, so the gradient at A is 3, and A = (2, 12)

Tangent: $y = 3x + 6$

Equate: $x^3 - 4x^2 + 4x = 0$, which has a repeated root x = 2

Sum of the roots = 4, so 2 + 2 + x = 4 and x = 0

y = 6

Answer: B (0, 6)

9 Bounds of a compound expression

p = 8.8 and r = 5.2, each correct to 1 decimal place.
q = 1.48, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom

Top: LB p − UB q = 8.75 − 1.485 = 7.265

Bottom: UB r = 5.25

7.265 ÷ 5.25 = 1.38

Answer: 1.38

10 Counting factors

$N = 2^5 \times 7$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply

(5 + 1) × (1 + 1) = 6 × 2 = 12

Answer: 12