Day 104 **IGCSE 10-a-day: Model Solutions**

1 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$.

Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} + \begin{pmatrix} 2 \\ 8 \end{pmatrix} - \begin{pmatrix} -6 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{8} \\ \mathbf{-2} \end{pmatrix}$

2 Square roots as powers

$y = 8x^{5/2} - 2\sqrt{x}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = 8x^{5/2} - 2x^{1/2}$

$8x^{5/2} \rightarrow 20x^{3/2}$, $-2x^{1/2} \rightarrow -x^{-1/2}$

$\frac{dy}{dx} = 20x^{3/2} - x^{-1/2}$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{20x^{3/2} - x^{-1/2}}$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 49°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle OAB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 49 = 98°$

(b) OA = OB (radii), so triangle OAB is isosceles

Angle OAB = $(180 - 98) \div 2 = 41°$

Answer: (a) 98° (b) 41°

4 Solving quadratics by factorising

Solve $12x^2 + 17x = -6$

Rearrange so that one side is 0: $12x^2 + 17x + 6 = 0$

Factorise: $(4x + 3)(3x + 2) = 0$

$4x + 3 = 0$ or $3x + 2 = 0$

$x = -\frac{3}{4}$ or $x = -\frac{2}{3}$

Answer: $\mathbf{x = -\frac{3}{4}}$ **or** $\mathbf{x = -\frac{2}{3}}$

5 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{16}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 2.

(b) Find the value of t when the temperature is falling at 1 °C per minute.

$T = 20 + 16t^{-1}$, so $\frac{dT}{dt} = -16t^{-2}$

When t = 2: $\frac{dT}{dt} = -4$

$-\frac{16}{t^2} = -1$, so $t^2 = 16$ and t = 4

Answer: (a) −4 °C/min (b) t = 4

6 Histograms

The histogram shows the ages of visitors to a museum.
(a) Work out the total number of visitors.
(b) Estimate the number of visitors with an age between 40 years and 55 years.

Frequency = frequency density × class width
20 to 30: 1 × 10 = 10
30 to 50: 0.6 × 20 = 12
50 to 60: 0.8 × 10 = 8
60 to 80: 1.2 × 20 = 24
80 to 100: 0.4 × 20 = 8
(a) Total = 62
(b) 40 to 50: 0.6 × 10 = 6
50 to 55: 0.8 × 5 = 4, so estimate = 10
Answer: (a) 62 (b) 10

7 Increasing or decreasing at a point

$y = x^3 + 3x^2 - 9x + 3$
At the point where $x = -2$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 3x^2 + 6x - 9$

When $x = -2$: $\frac{dy}{dx} = -9$

-9 is negative, so the curve is decreasing
Answer: decreasing (gradient -9)

8 Variation: percentage change

y is directly proportional to the square root of x.
x is increased by 125%.
Work out the percentage change in y.

x is multiplied by 2.25
y is multiplied by $\sqrt{2.25}$
Multiplier for y = 1.5
Percentage increase = 50%
Answer: 50% increase

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 4 cm and base radius 10 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{4}{10}$

10 h = 4(h + 9), so 6 h = 36 and h = 6

Large cone: $\frac{1}{3} \times \pi \times 10^2 \times 15 = 500\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 6 = 32\pi$

Frustum = $500\pi - 32\pi = 468\pi$ cm^3
Answer: 468π cm^3

10 Profit and loss

A trader buys a games console for £40.
It adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 10%.
Work out the percentage profit or loss on the original cost.

Marked price = £40 × 1.3 = £52
Sale price = £52 × 0.9 = £46.80
Profit = £6.80

Percentage = $\frac{6.8}{40} \times 100 = 17\%$

Answer: 17% profit