Day 103 **IGCSE 10-a-day: Model Solutions**

1 Fractions

Work out $(4\frac{3}{5} + 3\frac{1}{3}) \div 2\frac{1}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{23}{5} + \frac{10}{3} = \frac{119}{15}$

$\frac{119}{15} \div \frac{7}{3} = \frac{119}{15} \times \frac{3}{7} = \frac{17}{5}$

$= 3\frac{2}{5}$

Answer: $3\frac{2}{5}$

2 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is R = x(28 − 2x) and the cost is C = 21 + 4x.

(a) Show that the profit P = R − C is P = $-2x^2 + 24x - 21$.

(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 28x - 2x^2 - (21 + 4x) = -2x^2 + 24x - 21$

$\frac{dP}{dx} = -4x + 24 = 0$ gives $x = 6$

$\frac{d^2P}{dx^2} = -4 < 0$, so a maximum

$P = 51$

Answer: 6000 items, maximum profit £51000

3 Factorising fully

Factorise fully $2x^3 - 72x$

Take out the common factor 2x: $2x(x^2 - 36)$

$x^2 - 36$ is a difference of two squares: $(x - 6)(x + 6)$

Answer: $2x(x - 6)(x + 6)$

4 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 \\ 5 & -4 \end{pmatrix}$.

Work out $A^2 - B^2$.

$A^2 = \begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix} \times \begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$B^2 = \begin{pmatrix} -4 & 3 \\ 5 & -4 \end{pmatrix} \times \begin{pmatrix} -4 & 3 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 31 & -24 \\ -40 & 31 \end{pmatrix}$

$A^2 - B^2 = \begin{pmatrix} -31 & 24 \\ 40 & -31 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-31} & \mathbf{24} \\ \mathbf{40} & \mathbf{-31} \end{pmatrix}$

5 Tangents parallel to a line

A curve has equation $y = x^3 - 6x^2 + 14x - 1$.
Find the equations of the two tangents to the curve that are parallel to the line $y = 5x + 5$.

Parallel lines have equal gradients: gradient = 5

$\frac{dy}{dx} = 3x^2 - 12x + 14 = 5$, so $3x^2 - 12x + 9 = 0$

$3(x - 1)(x - 3) = 0$ gives $x = 1$ or $x = 3$

At $x = 1$: $y = 8$, tangent $y - 8 = 5(x - 1)$

At $x = 3$: $y = 14$, tangent $y - 14 = 5(x - 3)$

Answer: $y = 5x + 3$ and $y = 5x - 1$

6 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 4 cm, top radius 2 cm and base radius 4 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+4} = \frac{2}{4}$

$4h = 2(h + 4)$, so $2h = 8$ and $h = 4$

Large cone: $\frac{1}{3} \times \pi \times 4^2 \times 8 = \frac{128}{3}\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 4 = \frac{16}{3}\pi$

Frustum = $\frac{128}{3}\pi - \frac{16}{3}\pi = \frac{112}{3}\pi$ cm^3

Answer: $\frac{112}{3}\pi$ cm^3

7 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix}$.

(a) Find M^{-1}.
(b) Use your answer to (a) to solve
$-2x - y = -7$
$x + 4y = 7$

(a) $\det M = -7$ so $M^{-1} = -\frac{1}{7}\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -7 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Answer: (a) $-\frac{1}{7}\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}$ (b) $x = 3$, $y = 1$

8 Increasing range for y = ax + b/x

$y = 5x + \frac{20}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 5 - \frac{20}{x^2}$

$\frac{dy}{dx} > 0$: $5 > \frac{20}{x^2}$

$x^2 > 4$ (x^2 is positive, so multiply through)

$x < -2$ or $x > 2$

Answer: $x < -2$ or $x > 2$

Day 103 **IGCSE 10-a-day: Model Solutions**

9 Bounds of roots

The area of a square is 210 cm^2, correct to the nearest cm^2.
Work out the lower bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$209.5 \leq$ area < 210.5
Lower bound of side = $\sqrt{209.5}$
= 14.4741... = 14.5 (3 s.f.)
Answer: 14.5 cm

10 Probability and quadratic equations

A bag contains some sweets. 9 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{4}{17}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{9}{n} \times \frac{8}{n-1} = \frac{72}{n(n-1)}$

$\frac{72}{n(n-1)} = \frac{4}{17}$, so $n(n-1) = 306$, giving $n^2 - n - 306 = 0$
$(n-18)(n+17) = 0$, so $n = 18$ (n cannot be negative)
Number of sweets that are yellow = $18 - 9 = 9$
Answer: 9