Day 102 **IGCSE 10-a-day: Model Solutions**

1 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{5}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 4 days.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{1}{5})^4 = \frac{4}{5}^4 = 0.4096$

P(at least once) = 1 − P(never) = 1 − 0.4096
= 0.590

Answer: 0.590

2 Column vectors

The vectors $\begin{pmatrix}2k\\32\end{pmatrix}$ and $\begin{pmatrix}2\\2k\end{pmatrix}$ are parallel.

k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same

$\frac{2k}{2} = \frac{32}{2k}$

$2k \times 2k = 32 \times 2$

$4\,k^2 = 64$

$k^2 = 16$

k = 4 (k is positive)

Answer: k = 4

3 Multiplying 2 by 2 matrices

A = $\begin{pmatrix}2 & -4\\3 & -4\end{pmatrix}$ and B = $\begin{pmatrix}3 & 5\\3 & 1\end{pmatrix}$.

(a) Work out AB.
(b) Work out BA.

(a) AB: row 1 is 2 × 3 + (−4) × 3 and 2 × 5 + (−4) × 1; row 2 is 3 × 3 + (−4) × 3 and 3 × 5 + (−4) × 1

AB = $\begin{pmatrix}-6 & 6\\-3 & 11\end{pmatrix}$

(b) BA = $\begin{pmatrix}21 & -32\\9 & -16\end{pmatrix}$

Answer: (a) AB = $\begin{pmatrix}-6 & 6\\-3 & 11\end{pmatrix}$ (b) BA = $\begin{pmatrix}21 & -32\\9 & -16\end{pmatrix}$

4 Turning point of a quadratic by differentiation

$f(x) = x^2 - 6$

Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

f′(x) = 2x

f′(x) = 0: 2x = 0, so x = 0

f(0) = −6

f″(x) = 2, which is positive, so it is a minimum

Answer: (0, −6), minimum

5 Standard form

Work out $(3.75 \times 10^{-2}) \div (1.25 \times 10^{-1})$
Give your answer in standard form.

$3.75 \div 1.25 = 3$
$10^{-2} \div 10^{-1} = 10^{-1}$
$= 3 \times 10^{-1}$
Answer: 3×10^{-1}

6 Given a stationary point, find k

The curve $y = x^3 + 3x^2 + kx$ has a stationary point at $x = -3$.
(a) Find the value of k.
(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + 6x + k$
$x = -3$: $9 + k = 0$, so $k = -9$
$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x + 3)(x - 1)$
Other root: $x = 1$
$y = -5$
Answer: (a) $k = -9$ (b) $(1, -5)$

7 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 29$
$y = -x - 3$

Substitute into the first equation: $x^2 + (-x - 3)^2 = 29$
Expand: $2x^2 + 6x - 20 = 0$
Divide by 2: $x^2 + 3x - 10 = 0$
Factorise: $(x + 5)(x - 2) = 0$, so $x = -5$ or $x = 2$
$y = -x - 3$ gives $y = 2$ and $y = -5$
Answer: $x = -5$, $y = 2$ and $x = 2$, $y = -5$

8 Area of a segment

The circle has centre O and radius 5 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 5^2 = 30.543$ cm^2
Area of triangle OAB = $\frac{1}{2} \times 5 \times 5 \times \sin 140° = 8.0348$ cm^2
Segment = sector − triangle = $30.543 - 8.0348 = 22.508$
$= 22.5$ cm^2 (3 s.f.)
Answer: 22.5 cm^2

9 Bounds: finding an unknown length

A rectangle has area 120 cm^2, correct to 2 significant figures.
Its length is 14 cm, correct to 2 significant figures.
Work out the lower bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $115 \leq A < 125$. Length: $13.5 \leq l < 14.5$
LB width = LB area ÷ UB length = $115 \div 14.5 = 7.93$
Answer: 7.93 cm

10 Increasing for all x

$f(x) = x^3 + kx^2 + 3x - 4$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 3$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 3 < 0$, so $4k^2 < 36$
$k^2 < 9$, so $-3 < k < 3$
Answer: $-3 < k < 3$