Day 101 **IGCSE 10-a-day: Model Solutions**

1 Function values and composites

f(x) = 5x + 8 and g(x) = x^2 + 7.
Work out ff(−2) + gf(−2).

f(−2) = −2, so ff(−2) = f(−2) = −2
f(−2) = −2, so gf(−2) = g(−2) = 11
−2 + 11 = 9
Answer: 9

2 Where is the second derivative zero?

$y = x^3 - 6x^2 + 5x + 2$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 3x^2 - 12x + 5$
$\frac{d^2y}{dx^2} = 6x - 12$
$\frac{d^2y}{dx^2} = 0$ gives 6x − 12 = 0, so x = 2
$\frac{dy}{dx} = -7$ when x = 2
Answer: (a) 6x − 12 (b) x = 2, gradient −7

3 Sets of numbers: lists and counting

ξ = integers from 1 to 20
A = the numbers greater than 10
B = the square numbers
(a) List the elements of A ∩ B′.
(b) Find n(A ∪ B).

A = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20
B = 1, 4, 9, 16
(a) A ∩ B′ = 11, 12, 13, 14, 15, 17, 18, 19, 20
(b) A ∪ B = 1, 4, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 which has 13 elements
Answer: (a) 11, 12, 13, 14, 15, 17, 18, 19, 20 (b) 13

4 Inverse proportion in context

9 machines can make a batch of bolts in 10 hours.
After 4 hours, 6 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 9 × 10 = 90 machine-hours
Work done = 9 × 4 = 36, so work left = 54
Workers left = 9 − 6 = 3
Time = 54 ÷ 3 = 18 hours
Answer: 18 hours

5 Tangent where the curve crosses the y-axis

The curve $y = -x^2 + 2x + 2$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, x = 0 and y = 2
$\frac{dy}{dx} = -2x + 2$, so the gradient at A is 2
Tangent: y = 2x + 2
At B, y = 0: 2x + 2 = 0, so x = −1
Answer: (−1, 0)

6 Profit and loss

A retailer buys a printer for £230.
It adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 20%.
Work out the percentage profit or loss on the original cost.

Marked price = £230 × 1.3 = £299
Sale price = £299 × 0.8 = £239.20
Profit = £9.20

Percentage = $\frac{9.2}{230} \times 100 = 4\%$

Answer: 4% profit

7 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 12x + 35}{x^2 - 4x} \times \frac{x^2 + 2x}{x - 5}$

Factorise: $\frac{(x - 5)(x - 7)}{x(x - 4)} \times \frac{x(x + 2)}{(x - 5)}$

Cancel $(x - 5)$ and x

$= \frac{(x - 7)(x + 2)}{(x - 4)}$

Answer: $\frac{\mathbf{(x - 7)(x + 2)}}{\mathbf{(x - 4)}}$

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = $-\mathbf{a} + 4\mathbf{b}$, OQ = $-2\mathbf{a} + 6\mathbf{b}$ and OR = $-4\mathbf{a} + 10\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-\mathbf{a} + 2\mathbf{b}$
QR = OR − OQ = $-2\mathbf{a} + 4\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 2
Answer: (b) 1 : 2

9 Where two tangents meet

The curve C has equation $y = x^2 + 5x - 3$.
The points A and B on C have x coordinates −2 and 2.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x + 5$. At A: gradient 1, A = (−2, −9)
Tangent at A: $y = x - 7$. At B: gradient 9, B = (2, 11)
Tangent at B: $y = 9x - 7$
Equate: $x - 7 = 9x - 7$, so $x = 0$ and $y = -7$
Answer: T (0, −7)

10 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 62°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 62 = 124°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 124 = 56°
Answer: (a) 124° (b) 56°