Day 100 **IGCSE 10-a-day: Model Solutions**

1 Reverse percentages

The number of members of a club is decreased by 40% in one year.
The next year the number of members is increased by 12%.
The next year the number of members is increased by 25%.
There are now 168 members.
How many members were there to begin with?

Multipliers: 0.6, 1.12, 1.25
Overall multiplier = 0.6 × 1.12 × 1.25 = 0.84
Original = final ÷ multiplier = 168 ÷ 0.84
= 200
Answer: 200

2 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{16}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 4.

(b) Find the value of t when the temperature is falling at 1 °C per minute.

$T = 20 + 16t^{-1}$, so $\frac{dT}{dt} = -16t^{-2}$

When t = 4: $\frac{dT}{dt} = -1$

$-\frac{16}{t^2} = -1$, so $t^2 = 16$ and t = 4

Answer: (a) −1 °C/min (b) t = 4

3 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
40% of the juniors and 45% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
40% of 60 = 24 and 45% of 40 = 18
Together: 42 out of 100
42 ÷ 100 × 100 = 42%
Answer: 42%

4 Exact trigonometric values

Work out the exact value of $(\tan 45°)^2 + \sin 30°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\tan 45°)^2 + \sin 30° = (1)^2 + \frac{1}{2}$

$= \frac{3}{2}$

Answer: $\frac{3}{2}$

5 Equation of a normal

A curve has equation $y = x^2 - 2x - 1$.
Find the equation of the normal to the curve at the point where $x = 0$.

When $x = 0$, $y = -1$ and $\frac{dy}{dx} = -2$

Gradient of the normal $= -1 \div (-2) = \frac{1}{2}$

$y + 1 = \frac{1}{2}x$

$y = \frac{1}{2}x - 1$

Answer: $y = \frac{1}{2}x - 1$

6 Tangency conditions

The line $y = 4x + k$ is a tangent to the curve $y = 3x^2 + 2x$.
Find the value of k.

Equate: $3x^2 + 2x = 4x + k$
Rearrange: $3x^2 - 2x - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $4 - 4 \times 3 \times (-k) = 0$
$12k + 4 = 0$
$k = -\frac{1}{3}$

Answer: $k = -\frac{1}{3}$

7 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx + 6$ has stationary points at $x = -3$ and $x = -1$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = -3$: $27 + -6a + b = 0$. $x = -1$: $3 + -2a + b = 0$
Subtract the equations: $24 + -4a = 0$, so $a = 6$
Then $b = 9$
Answer: $a = 6$, $b = 9$

8 Exponential graph from points

The diagram shows part of the graph of $y = ab^x$.
The curve passes through (0, 24) and (2, 6).
(a) Find the values of a and b. (b) Work out the value of y when $x = 3$.

(a) At $x = 0$: $y = a \times b^0 = a$, so $a = 24$
At $x = 2$: $24 \times b^2 = 6$ so $b^2 = 0.25$
$b = 0.5$
(b) $y = 24 \times 0.5^3 = 3$
Answer: (a) $a = 24$, $b = 0.5$ (b) 3

9 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 60°
$\tan 60° = \sqrt{3}$
$x = 8 \times \tan 60° = 8 \times \sqrt{3}$
$x = 8\sqrt{3}$
Answer: $x = 8\sqrt{3}$

10 Solving AX = B

$A = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = -\frac{1}{3}\begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}$

$X = -\frac{1}{3}\begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

Answer: $\mathbf{X = \begin{pmatrix} -4 \\ 0 \end{pmatrix}}$