Day 10 **IGCSE 10-a-day: Model Solutions**

1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x + 10)°
Angle ABC = (x + 22)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x + 10) + 2(x + 22) = 180
6x + 54 = 180, so 6x = 126
x = 21
(b) Angle BAC = 4 × 21 + 10 = 94°
Answer: (a) x = 21 (b) 94°

2 Laws of indices with algebra

Simplify fully $\frac{8x^3y^5 \times 9x^6y}{8x^6y^2}$

Give your answer in the form kx^ay^b

Numbers: 8 × 9 ÷ 8 = 9
Powers of x: 3 + 6 − 6 = 3
Powers of y: 5 + 1 − 2 = 4
$= 9x^3y^4$
Answer: $9x^3y^4$

3 Range and inverse of a quadratic

$g(x) = x^2 + 4x + 1$ for $x \leq -2$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 2)^2 - 3$
$x \leq -2$ includes the minimum point, so $g(x) \geq -3$
(b) $y = (x + 2)^2 - 3$, so $(x + 2)^2 = y + 3$
$x \leq -2$ means $x + 2$ is negative or zero, so $x = -2 - \sqrt{y + 3}$
$g^{-1}(x) = -2 - \sqrt{x + 3}$
Answer: (a) $g(x) \geq -3$ (b) $g^{-1}(x) = -2 - \sqrt{x + 3}$

4 LCM problems

A church bell rings every 5 minutes.
Another bell rings every 6 minutes.
A third bell rings every 8 minutes.
They all ring together at 6:00 am.
How many times do they all ring together between 6:00 am and 3:00 pm, including both 6:00 am and 3:00 pm if they do?

Find the LCM of 5, 6 and 8
$5 = 5$, $6 = 2 \times 3$, $8 = 2^3$
LCM $= 2^3 \times 3 \times 5 = 120$
Total time from 6:00 am to 3:00 pm = 540 minutes
540 ÷ 120 = 4.5, so there are 4 more times after the start
Including the start: 4 + 1 = 5
Answer: 5

5 Tangent at a turning point

The graph of $y = x^2 - 2x + 2$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 2x - 2$

At P, $\frac{dy}{dx} = 0$, so $2x - 2 = 0$ and $x = 1$

$y = 1$ at P
The gradient is 0, so the tangent is horizontal: $y = 1$
Answer: y = 1

6 Counting subsets

The set A has 7 elements.
(a) Write down the number of subsets of A.
(b) Write down the number of proper subsets of A.

(a) Each element is either in or out of a subset: $2^7 = 128$
(b) Remove the set A itself: $128 - 1 = 127$
Answer: (a) 128 (b) 127

7 Rates of work

Tap A can fill an empty tank in 10 minutes.
Tap B can fill an empty tank in 15 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{10}$ and Tap B does $\frac{1}{15}$

Together: $\frac{1}{10} + \frac{1}{15} = \frac{25}{150}$

Time = $\frac{150}{25}$ = 6 minutes

Answer: 6 minutes

8 Stationary point of y = ax + b/√x

A curve has equation $y = 4x + \frac{8}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4 - 4x^{-3/2}$

$\frac{dy}{dx} = 0$: $4x^{-3/2} = 4$, so $x^{3/2} = 1$

$x = 1^{2/3} = 1$, and $y = 12$

$\frac{d^2y}{dx^2} = 6x^{-5/2}$

At $x = 1$, $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: (1, 12), minimum

9 Venn diagrams with algebra

The Venn diagram shows how many of 19 students study French (F) or Spanish (S).
(a) Work out the value of x.
(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: $(x + 1) + x + (2x - 1) + 7 = 19$
$4x + 7 = 19$, so $x = 3$
(b) Students in the second circle: $x + 2x - 1 = 3 + 5 = 8$, of whom 3 are in both
Probability = $\frac{3}{8}$

Answer: (a) $x = 3$ (b) $\frac{3}{8}$

10 Points with a given gradient (cubic)

The curve $y = 2x^3 + 12x^2 + 10x - 3$ has gradient -8 at two points.
Find the x coordinates of these points.

$\frac{dy}{dx} = 6x^2 + 24x + 10$
$6x^2 + 24x + 10 = -8$, so $6x^2 + 24x + 18 = 0$
Divide by 6: $x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$, so $x = -3$ or $x = -1$
Answer: $x = -3$ and $x = -1$