Day 1 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $2x^3 + 4x - 4$
(a) Find f′(x).
(b) Work out the value of f′(−1).

(a) $2x^3 \rightarrow 6x^2$, $4x \rightarrow 4$
f′(x) = $6x^2 + 4$
(b) f′(−1) = 10
Answer: (a) f′(x) = $6x^2 + 4$ (b) 10

2 Position vector of a point dividing a line

The position vectors of A and P are $\begin{pmatrix}-6\\1\end{pmatrix}$ and $\begin{pmatrix}-4\\1\end{pmatrix}$.
P lies on AB so that AP : PB = 2 : 3.
Find the position vector of B.

$\mathbf{AP} = \begin{pmatrix}-4\\1\end{pmatrix} - \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

AP is 2 parts, so each part is $\frac{1}{2}$ of $\mathbf{AP} = \begin{pmatrix}1\\0\end{pmatrix}$

AB is 5 parts: $\mathbf{AB} = 5 \times \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix}$

$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \begin{pmatrix}-1\\1\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-1}\\\mathbf{1}\end{pmatrix}$

3 Composite functions

f(x) = 4x + 2 and g(x) = $\frac{8}{x}$
(a) Find gf(x).
(b) Solve gf(x) = $\frac{4}{11}$

(a) gf(x) = g(4x + 2) = $\frac{8}{4x+2}$
(b) $\frac{8}{4x+2} = \frac{4}{11}$, so $4x + 2 = 8 \div \frac{4}{11} = 22$
4x = 20
x = 5
Answer: (a) $\frac{8}{4x+2}$ (b) x = 5

4 Fraction problems

A bath is $\frac{1}{10}$ full.
288 more litres of water are added and the bath is then $\frac{9}{10}$ full.
Work out the capacity of the bath.

Fraction added = $\frac{9}{10} - \frac{1}{10} = \frac{4}{5}$

$\frac{4}{5}$ of the capacity is 288 litres

Capacity = $288 \div \frac{4}{5} = 360$

Answer: 360 litres

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5 Turning points of a cubic

A curve has equation $y = 2x^3 - 12x^2 + 18x - 5$.
Find the coordinates of the turning points of the curve. Use the second derivative to determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 - 24x + 18 = 6(x - 1)(x - 3)$

$\frac{dy}{dx} = 0$ gives $x = 1$ or $x = 3$

$y = 3$ when $x = 1$ and $y = -5$ when $x = 3$

$\frac{d^2y}{dx^2} = 12x - 24$: at $x = 1$ it is -12 (maximum), at $x = 3$ it is 12 (minimum)

Answer: (1, 3) maximum, (3, −5) minimum

6 Tree diagrams and reverse conditional probability

75% of a town's people have been vaccinated.
5% of vaccinated people catch a virus and 3% of unvaccinated people catch it.
A person is chosen at random.
(a) Work out the probability that the person catches the virus.
(b) The person has caught the virus. Work out the probability that they were vaccinated.

(a) $0.75 \times 0.05 = 0.0375$ and $0.25 \times 0.03 = 0.0075$
P(faulty) = 0.0375 + 0.0075 = 0.045

(b) $\frac{0.0375}{0.045} = \frac{5}{6}$

Answer: (a) 0.045 (b) $\frac{5}{6}$

7 Shaded areas: exact answers

ABCD is a square of side 8 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 8) = $\frac{1}{4} \times \pi \times 8^2 = 16\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = $2 \times 16\pi - 64 = 32\pi - 64$

Answer: $(32\pi - 64)$ cm^2

8 Percentages of percentages

In a group of employees, 75% are men and the rest are women.
45% of the men and 20% of the women work from home.
What percentage of all the employees work from home?

Take 100 employees: 75 are men and 25 are women
45% of 75 = 33.75 and 20% of 25 = 5
33.75 + 5 = 38.75 out of 100

Answer: 38.75%

9 Percentage change and proportion

The time for a pendulum to swing is directly proportional to the square root of the length of the pendulum.
The length of the pendulum is increased by 69%.
Work out the percentage change in the time for a pendulum to swing.

The length of the pendulum is multiplied by 1.69

The multiplier for the time for a pendulum to swing is $\sqrt{1.69} = 1.3$
1.3 is an increase of 30%

Answer: 30% increase

10 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{64}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when $t = 4$.

(b) Find the value of t when the temperature is falling at 4 °C per minute.

$T = 20 + 64t^{-1}$, so $\frac{dT}{dt} = -64t^{-2}$

When $t = 4$: $\frac{dT}{dt} = -4$

$-\frac{64}{t^2} = -4$, so $t^2 = 16$ and $t = 4$

Answer: (a) −4 °C/min (b) t = 4