Day 206 **B Tier 10-a-day: Model Solutions**

1 Trigonometry: finding a side

Calculate the length of d. (Calculator)
Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Label the sides: d is the opposite and 16 m is the hypotenuse
We use sin (opposite and hypotenuse)

$\sin 40° = \frac{d}{16}$

$d = 16 \times \sin 40° = 10.285...$
d = 10.3 m (1 d.p.)
Answer: d = 10.3 m

2 Powers and roots

Work out
(a) $\sqrt[3]{343} \times 3$
(b) the square root of 9

(a) $\sqrt[3]{343} = 7$ because $7 \times 7 \times 7 = 343$, so $7 \times 3 = 21$
(b) $3 \times 3 = 9$, so the square root is 3
Answer: (a) 21 (b) 3

3 Map scales

The scale of a map is 1 : 100,000.
Two villages are 8 km apart.
How far apart are they on the map?
Give your answer in centimetres.

8 km = 800,000 cm
Divide by 100,000: 800,000 ÷ 100,000 = 8
Answer: 8 cm

4 Equations with fractions

Solve $\frac{x - 3}{5} = 3$

Multiply both sides by 5: $x - 3 = 15$
Add 3 to both sides
Answer: x = 18

5 LCM problems

Two buses leave a bus station at 09:15.
One bus leaves every 16 minutes.
The other bus leaves every 10 minutes.
At what time do both buses next leave together?

Multiples of 16: 16, 32, 48, 64, 80
Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80
The first common multiple is 80, so the LCM is 80 minutes
09:15 + 80 minutes = 10:35
Answer: 10:35

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6 Pythagoras' theorem

Calculate the length of x. (Calculator)
Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use Pythagoras: $a^2 + b^2 = c^2$, where c is the longest side
$x^2 + 7^2 = 10^2$
$x^2 = 100 - 49 = 51$
$x = \sqrt{51} = 7.141...$
x = 7.1 cm (1 d.p.)
Answer: x = 7.1 cm

7 Repeated percentage change

The price of a laptop is £160.
The price is first reduced by 5%, then the new price is reduced by 20%.
(a) Work out the final price.
(b) Work out the overall percentage decrease in the price.

(a) Multipliers: 0.95 and 0.8
160 × 0.95 × 0.8 = £121.60
(b) Overall multiplier = 0.76
Overall decrease = 24%
Answer: (a) £121.60 (b) 24% decrease

8 Compound interest

Maya invests £2,000 in an account paying 6% compound interest per year. (Calculator)
Work out the value of her investment after 2 years.

Multiplier = 1 + 6 ÷ 100 = 1.06
Value = $2{,}000 \times 1.06^2$
= £2,247.20
Answer: £2,247.20

9 Iteration

Here is an iteration formula.
$x_{n+1} = \sqrt{3x_n + 1}$
$x_0 = 5$
(a) Work out x_1
(b) Work out x_2. Give your answer correct to 3 decimal places. (Calculator)

(a) $x_1 = \sqrt{3 \times 5 + 1} = \sqrt{16} = 4$
(b) $x_2 = \sqrt{3 \times 4 + 1} = \sqrt{13} = 3.60555...$
= 3.606
Answer: (a) 4 (b) 3.606

10 Capture-recapture

40 fish in a lake are caught, tagged and returned to the lake.
Later, a sample of 60 fish is caught and 10 of them are tagged.
Estimate the total number of fish in the lake.

Proportion tagged in the sample = 10 ÷ 60
This is the same proportion as 40 out of the whole population
Population ≈ 40 × 60 ÷ 10 = 240
Answer: 240