Day 156 **B Tier 10-a-day: Model Solutions**

1 Simplifying ratios

Write the ratio 16 : 24 in its simplest form.

The highest common factor of 16 and 24 is 8
16 ÷ 8 = 2 and 24 ÷ 8 = 3
Answer: 2 : 3

2 Forming equations

I think of a number.
I multiply it by 6 and then subtract 10.
The answer is 62.
Work out my number.

Let the number be n: 6n − 10 = 62
Add 10: 6n = 72
Divide by 6
Answer: 12

3 Standard form

Write each number as an ordinary number.
(a) 3.7×10^{6}
(b) 7.6×10^{-5}

(a) Multiply by 10, 6 times: move the decimal point 6 places right
$3.7 \times 10^{6} = 3{,}700{,}000$
(b) $10^{-5} = 0.00001$, so move the decimal point 5 places left
$7.6 \times 10^{-5} = 0.000076$
Answer: (a) 3,700,000 (b) 0.000076

4 Scale drawings and maps

On a map, 1 cm represents 10 km.
Two towns are 2.5 cm apart on the map.
(a) Work out the real distance between the towns.
(b) Write the scale of the map in the form 1 : n.

(a) 2.5 × 10 = 25 km
(b) 10 km = 1 000 000 cm
So 1 cm on the map is 1 000 000 cm in real life
Answer: (a) 25 km (b) 1 : 1 000 000

5 Angles and algebra

The angles of a triangle are x°, 3x° and (3x + 40)°.
Work out the size of the largest angle.

Angles in a triangle add up to 180°
7x + 40 = 180
7x = 140, so x = 20
The angles are 20°, 60°, 100°
Answer: 100°

6 Population density

City E has a population of 7,500.
Its area is 50 km^2.
Work out the population density.
Give your answer in people per km^2.

Population density = population ÷ area
7,500 ÷ 50 = 150
Answer: 150 people per km^2

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7 Trigonometry in context

A lighthouse is 29 m tall. (Calculator)
Mia stands 46 m from the base of the lighthouse on level ground.
Work out the angle of elevation of the top of the lighthouse from Mia.
Give your answer correct to 1 decimal place.

The height is the opposite side and the distance is the adjacent side

$\tan\theta = \frac{29}{46}$

$\theta = \tan^{-1}(\frac{29}{46}) = 32.229...$

$\theta = 32.2°$ (1 d.p.)

Answer: 32.2°

8 LCM problems

Two lights flash at 07:00.
One light flashes every 30 minutes.
The other light flashes every 25 minutes.
At what time do both lights next flash together?

Multiples of 30: 30, 60, 90, 120, 150
Multiples of 25: 25, 50, 75, 100, 125, 150
The first common multiple is 150, so the LCM is 150 minutes
07:00 + 150 minutes = 09:30

Answer: 09:30

9 Iteration

Here is an iteration formula.

$x_{n+1} = \sqrt{3x_n + 1}$

$x_0 = 1$

(a) Work out x_1

(b) Work out x_2. Give your answer correct to 3 decimal places. (Calculator)

(a) $x_1 = \sqrt{3 \times 1 + 1} = \sqrt{4} = 2$

(b) $x_2 = \sqrt{3 \times 2 + 1} = \sqrt{7} = 2.64575...$

$= 2.646$

Answer: (a) 2 (b) 2.646

10 Bearings with Pythagoras and trigonometry

A boat sails 8 km on a bearing of 090° from P to Q, then 15 km on a bearing of 180° to R. (Calculator)
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give your answers correct to 1 decimal place.

(a) Triangle PQR has a right angle at Q

$PR^2 = 8^2 + 15^2 = 289$

$PR = \sqrt{289} = 17$ km

(b) tan (angle QPR) = $\frac{15}{8}$, so angle QPR = 61.928° (to 3 d.p.)

PQ is on a bearing of 090°, and R is a further 61.928° clockwise from PQ
Bearing = 90 + 61.928 = 151.9°

Answer: (a) 17 km (b) 151.9°