Day 103 **B Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line

A is the point $(-2, 3)$ and B is the point $(4, 8)$. (Calculator)
(a) Work out the coordinates of the midpoint of AB.
(b) Work out the length of AB. Give your answer correct to 1 decimal place.

(a) Midpoint = $(2 \div 2, 11 \div 2) = (1, 5.5)$
(b) Horizontal distance = 6, vertical distance = 5
$AB^2 = 6^2 + 5^2 = 36 + 25 = 61$
$AB = \sqrt{61} = 7.810...$
Answer: (a) (1, 5.5) (b) 7.8

2 HCF and LCM

Find the (a) highest common factor (HCF), (b) lowest common multiple (LCM) of 36 and 84.

$36 = 2^2 \times 3^2$ and $84 = 2^2 \times 3 \times 7$
(a) HCF: multiply the primes in both lists, using the smaller power: $2^2 \times 3 = 12$
(b) LCM: use the larger power of every prime: $2^2 \times 3^2 \times 7 = 252$
Answer: (a) 12 (b) 252

3 Simplifying expressions

Simplify fully
(a) $9m \times 7n$
(b) $10p^2 \div 2p$

(a) Multiply the numbers: $9 \times 7 = 63$, then the letters: $m \times n = mn$
(b) $10 \div 2 = 5$ and $p^2 \div p = p$
Answer: (a) 63mn (b) 5p

4 Stratified sampling

A stratified sample of 40 students is taken from the 400 students in the table.
Work out how many students should be sampled from Year 7.

Sample fraction = $40 \div 400$
Year 7: $80 \div 400 \times 40 = 8$
Answer: 8

5 Column vectors

$\mathbf{a} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}$
Work out
(a) $2\mathbf{a} - \mathbf{b}$
(b) $3\mathbf{a} + 2\mathbf{b}$

(a) $\begin{pmatrix} -10 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ -6 \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$
(b) $\begin{pmatrix} -15 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -12 \end{pmatrix} = \begin{pmatrix} -17 \\ -15 \end{pmatrix}$
Answer: (a) $\begin{pmatrix} \mathbf{-9} \\ \mathbf{4} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{-17} \\ \mathbf{-15} \end{pmatrix}$

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6 Forming and solving equations

A triangle has sides of length x cm, (x + 3) cm and (2x − 1) cm.
The perimeter of the triangle is 22 cm.
(a) Work out the value of x.
(b) Work out the length of the longest side.

(a) x + (x + 3) + (2x − 1) = 22
4x + 2 = 22
4x = 20, so x = 5
(b) The sides are 5 cm, 8 cm and 9 cm
Answer: (a) x = 5 (b) 9 cm

7 Estimation

Estimate the value of

$\frac{5.7}{4.77} + 0.493$

Show your rounding to 1 significant figure.

Round each number to 1 significant figure: 5.7 → 6, 0.493 → 0.5, 4.77 → 5
Estimate = 6 ÷ 5 + 0.5 = 1.7
Answer: 1.7

8 Circle theorems

A, B, C and D are points on a circle.
Angle ADC = 76°.
Work out the size of angle ABC, marked p.
Give a reason for your answer.
The diagram is not drawn accurately.

ABCD is a cyclic quadrilateral
Opposite angles of a cyclic quadrilateral add up to 180°
p = 180 − 76 = 104°
Answer: p = 104°

9 Relative frequency

A coin is thrown 500 times.
It lands on heads 230 times.
(a) Work out the relative frequency of heads.
(b) Is the coin fair? Give a reason.

(a) 230 ÷ 500 = 0.46
(b) A fair coin should give about 0.5
0.46 is close to 0.5
Answer: (a) 0.46 (b) Probably fair, relative frequency close to 0.5

10 Iteration

Here is an iteration formula.

$x_{n+1} = 1 + \frac{15}{x_n}$

$x_0 = 5$

(a) Work out x_1
(b) Work out x_2. Give your answer correct to 3 decimal places. (Calculator)

(a) $x_1 = 1 + \frac{15}{5} = 1 + 3 = 4$

(b) $x_2 = 1 + \frac{15}{4} = 4.75...$

= 4.750
Answer: (a) 4 (b) 4.750