Day 98 **A Tier 10-a-day: Model Solutions**

1 Ratio and a difference

The ratio of red pens to blue pens in a box is 5 : 9.
There are 44 more blue pens than red pens.
Work out the number of red pens.

Difference in parts = 9 − 5 = 4
4 parts = 44, so 1 part = 44 ÷ 4 = 11
red pens: 5 × 11 = 55
Answer: 55

2 Dividing fractions in context

A jug holds $1\frac{3}{4}$ litres of juice. Glasses hold $\frac{1}{8}$ litre each.
How many glasses can be filled completely?

Divide: $1\frac{3}{4} \div \frac{1}{8}$

As improper fractions and multiplying by the reciprocal: $\frac{7}{4} \times 8$

= 14
Answer: 14

3 Box plots

The box plot shows the scores of the students in Class A in a test.
(a) Write down the median and the interquartile range.
For Class B the median is 50 and the interquartile range is 45.
(b) Compare the distributions of the two classes.

(a) Median = 40
Interquartile range = 55 − 20 = 35
(b) The median for Class B is higher (50 compared with 40), so it has the higher values on average
The IQR for Class A is smaller (35 compared with 45), so its values are more consistent
Answer: (a) Median 40, IQR 35 (b) Class B has the higher median; Class A has the smaller IQR so is more consistent

4 Terminating and recurring decimals

Here are four fractions: $\frac{24}{45}$ $\frac{39}{120}$ $\frac{1}{2}$ $\frac{6}{20}$
Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{24}{45} = \frac{8}{15}$, $\frac{39}{120} = \frac{13}{40}$, $\frac{1}{2}$, $\frac{6}{20} = \frac{3}{10}$
A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.
$\frac{8}{15}$ has denominator 15 = 3 × 5, which includes a factor other than 2 and 5.
Answer: $\frac{24}{45}$

5 The quadratic formula

Solve $4x^2 - x - 7 = 0$
Give your solutions correct to 2 decimal places. (Calculator)

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with a = 4, b = −1, c = −7
$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 4 \times (-7)}}{8}$
$x = \frac{1 \pm \sqrt{113}}{8}$
x = 1.45 or x = −1.20
Answer: x = 1.45 or x = −1.20

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6 Expanding with surds

Expand and simplify $(4 - 3\sqrt{2})^2$

Give your answer in the form $a + b\sqrt{c}$

$(4 - 3\sqrt{2})(4 - 3\sqrt{2}) = 16 - 12\sqrt{2} - 12\sqrt{2} + 18$

Collect: $34 - 24\sqrt{2}$

Answer: $34 - 24\sqrt{2}$

7 Expanding three brackets

Expand and simplify

$(x + 1)(x - 1)(x + 4)$

Expand the first two brackets: $(x + 1)(x - 1) = x^2 - 1$

Multiply by the third bracket: $(x^2 - 1)(x + 4)$

$= x^3 - x + 4x^2 - 4$

$= x^3 + 4x^2 - x - 4$

Answer: $x^3 + 4x^2 - x - 4$

8 Pythagoras in three dimensions

The diagram shows a cuboid.

Work out the length of the diagonal AG. (Calculator)

Give your answer to 3 significant figures.

The diagram is not drawn accurately.

$AG^2 = 8^2 + 12^2 + 11^2$

$= 64 + 144 + 121 = 329$

$AG = \sqrt{329} = 18.1$

Answer: 18.1 cm

9 Inverse proportion formulae

y is inversely proportional to x^2.

When x = 8, y = 2.

(a) Find a formula for y in terms of x.

(b) Work out the positive value of x when y = 5.12.

(a) $y = \frac{k}{x^2}$. Substitute: $2 = \frac{k}{64}$, so $k = 2 \times 64 = 128$

$y = \frac{128}{x^2}$

(b) $5.12 = \frac{128}{x^2}$, so $x^2 = 128 \div 5.12 = 25$, so $x = 5$

Answer: (a) $y = \frac{128}{x^2}$ (b) $x = 5$

10 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.

The outer radius is 4 cm and the inner radius is 3 cm.

The pipe is 15 cm long.

Work out the volume of metal used. Give your answer in terms of π.

Area of the ring $= \pi \times 4^2 - \pi \times 3^2 = \pi(16 - 9) = 7\pi$

Volume = area × length $= 7\pi \times 15$

$= 105\pi$

Answer: 105π cm^3