Day 96 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 4 flapjacks.
Butter: 40 g
Water: 100 ml
Oats: 100 g
(a) Work out how much butter is needed for 15 flapjacks.
(b) Hannah has 784 ml of water. What is the greatest number of flapjacks she can make?

(a) butter for one: 40 ÷ 4 = 10 g
For 15: 10 × 15 = 150 g
(b) water for one: 100 ÷ 4 = 25 ml
784 ÷ 25 = 31 remainder 9, so she can make 31
Answer: (a) 150 g (b) 31

2 Circle area and circumference

A circle has diameter 6 cm.
Work out, in terms of π,
(a) the circumference,
(b) the area.

(a) Circumference = $\pi d = \pi \times 6 = 6\pi$
(b) Area = $\pi r^2 = \pi \times 3^2 = 9\pi$
Answer: (a) 6π cm (b) 9π cm^2

3 Simultaneous equations

Solve the simultaneous equations
$-6x - 3y = 6$
$3x - y = -8$

Multiply (2) by 3 to make the y coefficients match
$-6x - 3y = 6$ (1)
$9x - 3y = -24$ (2)
The y terms have the same sign, so subtract equation (1) from equation (2)
$15x = -30$, so $x = -2$
Substitute $x = -2$ into equation (1) to find $y = 2$
Answer: $x = -2$, $y = 2$

4 Congruent triangles

Triangle XYZ: XY = 11 cm, YZ = 14 cm, angle XYZ = 85°.
Triangle DEF: DE = 14 cm, EF = 11 cm, angle DEF = 85°.
Are the two triangles congruent?
Give a reason for your answer.

XY = EF = 11 cm
YZ = DE = 14 cm
The equal angles are between these sides
Two sides and the included angle are equal, so the triangles are congruent (SAS)
Answer: Yes, congruent (SAS)

5 Successive percentage changes

The price of a product increases by 5% each year for 2 years.
Work out the overall percentage increase over the 2 years.

Multiplier for each year = 1.05
Overall multiplier = $1.05^2 = 1.1025$
Percentage increase = 10.25%
Answer: 10.25%

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6 Describing transformations

Describe fully the single transformation that maps shape A onto shape B.

The shape has been turned, so this is a rotation
Compare the matching vertices to find the angle and direction: a quarter turn clockwise
The centre is the point that stays fixed: (2, 2)
Answer: Rotation, 90° clockwise, centre (2, 2)

7 Composite functions

$f(x) = 4x + 2$
$g(x) = x^2 - 5$
Solve $fg(x) = -2$

$fg(x) = 4(x^2 - 5) + 2 = 4x^2 - 18$
$4x^2 - 18 = -2$, so $4x^2 = 16$
$x^2 = 4$, so $x = \pm 2$
Answer: $x = 2$ or $x = -2$

8 Reverse compound change

The value of a painting increases by 20% each year. (Calculator)
After 3 years it is worth £42,336.
Work out its value at the start.

Yearly multiplier = 1.2
Overall multiplier = $1.2^3 = 1.728$
Original value = £42,336 ÷ 1.728 = £24,500
Answer: £24,500

9 Mixing in a ratio

Jug A contains 15 litres of a mixture of orange juice and lemonade in the ratio 1 : 2.
Jug B contains 13 litres of a mixture of orange juice and lemonade in the ratio 7 : 6.
The contents of both jugs are mixed together.
Work out the ratio of orange juice to lemonade in the mixture. Give your answer in its simplest form.

Jug A: orange juice = 1 × 15 ÷ 3 = 5 litres, lemonade = 10 litres
Jug B: orange juice = 7 litres, lemonade = 6 litres
Total: orange juice = 12 litres, lemonade = 16 litres
Ratio = 12 : 16 = 3 : 4
Answer: 3 : 4

10 Finding a rate of compound change

Femi invests £16,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £19,965.
Work out the annual rate of interest.

Overall multiplier = £19,965 ÷ £16,500 = 1.21
Take the square root: $\sqrt{1.21} = 1.1$
Rate = (1.1 − 1) × 100 = 10%
Answer: 10%