Day 92 **A Tier 10-a-day: Model Solutions**

1 Similar shapes: finding a length

The two triangles are similar.
Work out the value of x.
The diagrams are not drawn accurately.

Scale factor from the small triangle to the large triangle = 21 ÷ 7 = 3
x = 6 × 3 = 18
Answer: x = 18 cm

2 Median and interquartile range

Here are the times, in seconds, taken by 11 runners.
22 6 40 45 34 52 36 58 43 28 39
(a) Work out the median.
(b) Work out the interquartile range.

Put in order: 6, 22, 28, 34, 36, 39, 40, 43, 45, 52, 58
(a) The middle value is number 6, so the median is 39
(b) Lower half: 6, 22, 28, 34, 36, so LQ = 28
Upper half: 40, 43, 45, 52, 58, so UQ = 45
IQR = 45 − 28 = 17
Answer: (a) 39 (b) 17

3 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 42°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 42 = 48
Answer: x = 48°

4 Equation of a straight line

A straight line passes through the points (3, −5) and (7, −1).
Find the equation of the line.

Gradient = $\frac{-1-(-5)}{7-3} = \frac{4}{4} = 1$
Use y = mx + c with m = 1 and the point (3, −5): −5 = 1 × 3 + c
c = −8
y = x − 8
Answer: y = x − 8

5 Successive percentage changes

The value of a painting is decreased by 5%.
The new value is then increased by 10%.
Work out the overall percentage change.

First change: multiplier 0.95. Second change: multiplier 1.1
Overall multiplier = 0.95 × 1.1 = 1.045
1.045 − 1 = 0.045, so the overall change is 4.5%
Answer: 4.5% increase

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6 Bounds of a speed or density

A runner travels 370 m in 12.9 s. The distance is measured to the nearest 10 metres and the time to the nearest 0.1 second. (Calculator)
Work out the upper bound for the runner's speed in m/s.
Give your answer correct to 3 significant figures.

speed = distance ÷ time
For the upper bound of speed use the upper bound of distance and the lower bound of time.
375 ÷ 12.85 = 29.18288
= 29.2 m/s (3 s.f.)
Answer: 29.2 m/s

7 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 40°.
Work out the size of angle x (angle OAB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
Angle AOB = 360 − 90 − 90 − 40 = 140°
OA = OB (radii), so triangle OAB is isosceles
x = (180 − 140) ÷ 2 = 20
Answer: x = 20°

8 Expanding three brackets

Expand and simplify
$(x + 3)(x + 1)(x - 2)$

Expand the first two brackets: $(x + 3)(x + 1) = x^2 + 4x + 3$
Multiply by the third bracket: $(x^2 + 4x + 3)(x - 2)$
$= x^3 + 4x^2 + 3x - 2x^2 - 8x - 6$
$= x^3 + 2x^2 - 5x - 6$
Answer: $x^3 + 2x^2 - 5x - 6$

9 Bounds and accuracy

A cyclist travels 700 metres in 6.5 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 705 ÷ 6.45 = 109.3023
Lower bound = 695 ÷ 6.55 = 106.1069
(b) Both bounds round to 110 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 110 m/s to 2 significant figures.
Answer: (a) 109.3023, 106.1069 (b) 110 m/s (2 s.f.)

10 Frequency density

The table gives information about 115 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $65 < l \leq 80$.

(a) Missing frequency = 115 − (40 + 21 + 6) = 48
(b) Class width = 80 − 65 = 15
Frequency density = 6 ÷ 15 = 0.4
Answer: (a) 48 (b) 0.4