Day 90 **A Tier 10-a-day: Model Solutions**

1 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.
The outer radius is 7 cm and the inner radius is 4 cm.
The pipe is 50 cm long.
Work out the volume of metal used. Give your answer to 3 significant figures. (Calculator)

Area of the ring = $\pi \times 7^2 - \pi \times 4^2 = \pi(49 - 16) = 33\pi$
Volume = area × length = $33\pi \times 50$
= 5180

Answer: 5180 cm^3

2 Direct proportion

Here are the ingredients for 12 cupcakes.
Sugar: 180 g
Butter: 180 g
Flour: 240 g
(a) Work out how much sugar is needed for 36 cupcakes.
(b) Hannah has 575 g of butter. What is the greatest number of cupcakes she can make?

(a) sugar for one: 180 ÷ 12 = 15 g
For 36: 15 × 36 = 540 g
(b) butter for one: 180 ÷ 12 = 15 g
575 ÷ 15 = 38 remainder 5, so she can make 38
Answer: (a) 540 g (b) 38

3 Profit and loss

Ryan buys a watch for £200 and sells it for £230.
Work out Ryan's percentage profit.

Profit = £230 − £200 = £30

Percentage profit = $\frac{30}{200} \times 100 = 15\%$

Answer: 15%

4 Difference of two squares

(a) Factorise $x^2 - 64$
(b) Use the same idea to work out $7^2 - 3^2$ without a calculator.

(a) Difference of two squares: $x^2 - 64 = x^2 - 8^2$
$(x - 8)(x + 8)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $7^2 - 3^2 = (7 + 3)(7 - 3)$
$= 10 \times 4 = 40$
Answer: (a) $(x - 8)(x + 8)$ (b) 40

5 Perimeter and area with algebra

A rectangle has length $(3x + 3)$ cm and width $(x - 2)$ cm.
The perimeter of the rectangle is 42 cm.
(a) Work out the value of x.
(b) Work out the area of the rectangle.

(a) $2 \times [(3x + 3) + (x - 2)] = 42$
$(3x + 3) + (x - 2) = 21$
$4x + 1 = 21$
$4x = 20$
$x = 5$
(b) Length = 18 cm and width = 3 cm
Area = 18 × 3 = 54
Answer: (a) x = 5 (b) 54 cm^2

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6 Algebraic proof

Prove that the sum of two consecutive multiples of 8 is always an odd multiple of 8.

Let the multiples be 8n and 8(n + 1)
Sum = 8n + 8n + 8 = 16n + 8
= 8(2n + 1)
2n + 1 is odd, so the sum is an odd multiple of 8
Answer: Proved

7 The sine rule

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$$\frac{\sin x}{8} = \frac{\sin 64°}{9}$$

$$\sin x = \frac{8 \times \sin 64°}{9}$$

$x = \sin^{-1}(0.7989)$
$x = 53.0°$
Answer: x = 53.0°

8 Rationalising with conjugates

Rationalise the denominator of $\frac{21}{3 + \sqrt{2}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 - \sqrt{2}$: $\frac{21(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

Denominator: $9 - 2 = 7$

$$\frac{21(3 - \sqrt{2})}{7} = 3(3 - \sqrt{2})$$

$= 9 - 3\sqrt{2}$
Answer: $9 - 3\sqrt{2}$

9 Exchange rates

£1 = €1.4.
(a) Change £320 to euros.
(b) Change €336 to pounds.

(a) 320 × 1.4 = 448
(b) 336 ÷ 1.4 = 240
Answer: (a) €448 (b) £240

10 Finding a rate of compound change

Leah invests £8,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £9,017.65.
Work out the annual rate of interest.

Overall multiplier = £9,017.65 ÷ £8,500 = 1.0609
Take the square root: $\sqrt{1.0609} = 1.03$
Rate = (1.03 − 1) × 100 = 3%
Answer: 3%