Day 89 **A Tier 10-a-day: Model Solutions**

1 Completing the square

Solve $x^2 + 8x + 4 = 0$ by completing the square.
Give your answers in surd form.

$x^2 + 8x + 4 = (x + 4)^2 - 12$
$(x + 4)^2 = 12$
$x + 4 = \pm\sqrt{12}$
$x = -4 \pm \sqrt{12} = -4 \pm 2\sqrt{3}$
Answer: $x = -4 \pm 2\sqrt{3}$

2 Independent events

The probability that Omar scores a goal is 0.1.
The probability that Grace scores a goal is 0.7.
The two events are independent.
Work out the probability that neither Omar nor Grace will score a goal.

P(Omar does not) = 1 − 0.1 = 0.9 and P(Grace does not) = 1 − 0.7 = 0.3
0.9 × 0.3 = 0.27
Answer: 0.27

3 Loci and constructions

XY is a straight line of length 8 cm.
A point S moves so that it is always the same distance from X and from Y.
Describe the locus of S.

Points the same distance from two fixed points lie on the perpendicular bisector
This line cuts XY in half at right angles
Answer: The perpendicular bisector of XY

4 Compound interest

Nadia invests £4,500 in an investment bond. (Calculator)
The account pays compound interest at 4% per year.
Work out the total interest earned after 5 years.

Multiplier = 1 + 4 ÷ 100 = 1.04
£4,500 × 1.04^5 = £5,474.94
Interest = £5,474.94 − £4,500 = £974.94
Answer: £974.94

5 The quadratic formula

Solve $2x^2 - 6x - 7 = 0$
Give your solutions correct to 2 decimal places. (Calculator)

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with a = 2, b = −6, c = −7

$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 2 \times (-7)}}{4}$

$x = \frac{6 \pm \sqrt{92}}{4}$

x = 3.90 or x = −0.90
Answer: x = 3.90 or x = −0.90

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6 Combining ratios

a : b = 1 : 2
b : c = 1 : 4
(a) Work out the ratio a : b : c
(b) a, b and c add up to 44. Work out the value of c.

(a) Make the b parts equal: lowest common multiple of 2 and 1 is 2
a : b = 1 : 2 and b : c = 2 : 8
a : b : c = 1 : 2 : 8
(b) 1 + 2 + 8 = 11 parts, so 1 part = 44 ÷ 11 = 4
c = 8 × 4 = 32
Answer: (a) 1 : 2 : 8 (b) 32

7 Parallel and perpendicular lines

The line L has equation $y = \frac{2}{3}x - 5$
Find the equation of the line that is perpendicular to L and passes through the point $(0, -2)$.

The gradient of L is $\frac{2}{3}$

Perpendicular gradient = $-1 \div \frac{2}{3} = -\frac{3}{2}$

$y = -\frac{3}{2}x + c$, so $-2 = -\frac{3}{2} \times 0 + c$

$c = -2$

$y = -\frac{3}{2}x - 2$

Answer: $y = -\frac{3}{2}x - 2$

8 Exact trigonometric values

Without using a calculator, work out the exact value of
2 cos 45° + 2 tan 30°
Give your answer in its simplest form.

$\cos 45° = \frac{\sqrt{2}}{2}$ and $\tan 30° = \frac{\sqrt{3}}{3}$

$2 \times \frac{\sqrt{2}}{2} + 2 \times \frac{\sqrt{3}}{3} = \sqrt{6}$

Answer: $\sqrt{6}$

9 Expanding with surds

Expand and simplify $(6 - 2\sqrt{7})^2$
Give your answer in the form $a + b\sqrt{c}$

$(6 - 2\sqrt{7})(6 - 2\sqrt{7}) = 36 - 12\sqrt{7} - 12\sqrt{7} + 28$
Collect: $64 - 24\sqrt{7}$
Answer: $64 - 24\sqrt{7}$

10 Bearings with the cosine rule

Ship A sails 24 miles from a port on a bearing of 265°.
Ship B sails 10 miles from the same port on a bearing of 300°. (Calculator)
Work out the distance between the two ships.
Give your answer to 3 significant figures.

The angle between the two paths = $|265° - 300°| = 35°$
$d^2 = 24^2 + 10^2 - 2 \times 24 \times 10 \times \cos 35°$
$d^2 = 282.81$
$d = 16.8$
Answer: 16.8 miles