Day 88 **A Tier 10-a-day: Model Solutions**

1 Two-way tables

The table shows information about 57 people.
One person is chosen at random.
(a) Find the probability that the person is a girl and studies Spanish.
(b) The person studies French. Find the probability that they are a boy.

(a) Girls who studies Spanish: 17 out of 57 = $\frac{17}{57}$

(b) French total = 35, of whom 30 are in the Boys row: $\frac{6}{7}$

Answer: (a) $\frac{17}{57}$ (b) $\frac{6}{7}$

2 Volume of a prism

The diagram shows a prism with a rectangle as its cross-section.
The volume of the prism is 1694 cm^3.
Work out the value of x.
The diagram is not drawn accurately.

Area of cross-section = 11 × 11 = 121
Volume = area × length
1694 = 121 × length
length = 1694 ÷ 121 = 14
Answer: x = 14

3 Pressure

A machine exerts a force of 100 N on the floor.
The pressure on the floor is 400 N/m^2.
Work out the area of the machine in contact with the floor.

Area = force ÷ pressure
= 100 ÷ 400 = 0.25 m^2
Answer: 0.25 m^2

4 Expanding and simplifying

Expand and simplify
$4(x + 5)(x + 4) - (x - 2)(x + 2)$

$4(x + 5)(x + 4) = 4(x^2 + 9x + 20) = 4x^2 + 36x + 80$
$(x - 2)(x + 2) = x^2 - 4$
Subtract: $3x^2 + 36x + 84$
Answer: $3x^2 + 36x + 84$

5 Column vectors

$\mathbf{a} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.
Work out $4\mathbf{a} + 1\mathbf{b}$.

$4\mathbf{a} = \begin{pmatrix} -12 \\ 20 \end{pmatrix}$ and $1\mathbf{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$

$4\mathbf{a} + 1\mathbf{b} = \begin{pmatrix} -12 + (-5) \\ 20 + 4 \end{pmatrix}$

$= \begin{pmatrix} -17 \\ 24 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-17} \\ \mathbf{24} \end{pmatrix}$

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6 Dividing fractions in context

A jug holds $11\frac{3}{5}$ litres of juice. Glasses hold $\frac{2}{5}$ litre each.

How many glasses can be filled completely?

Divide: $11\frac{3}{5} \div \frac{2}{5}$

As improper fractions and multiplying by the reciprocal: $\frac{58}{5} \times \frac{5}{2}$

= 29

Answer: 29

7 Bearings and distances

A plane leaves an airport and travels 9 miles on a bearing of 120°. (Calculator)
It then travels 12 miles on a bearing of 210°.
(a) Work out how far it is now from the airport.
(b) Work out the bearing of its final position from the airport. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle

(a) Distance^2 = 9^2 + 12^2 = 225, so distance = 15 miles

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{12}{9})$ = 53.1°

Bearing = 120° + 53.1° = 173.1°
Bearing = 173°

Answer: (a) 15 miles (b) 173°

8 Rationalising with conjugates

Rationalise the denominator of $\frac{54}{5-\sqrt{7}}$

Give your answer in the form a + b$\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate 5 + $\sqrt{7}$: $\frac{54(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})}$

Denominator: 25 − 7 = 18

$\frac{54(5+\sqrt{7})}{18}$ = 3(5 + $\sqrt{7}$)

= 15 + 3$\sqrt{7}$

Answer: 15 + 3$\sqrt{7}$

9 Reverse compound change

The value of an investment grows by 20% each year. (Calculator)
After 3 years it is worth £61,344.
Work out its value at the start.

Yearly multiplier = 1.2

Overall multiplier = 1.2^3 = 1.728

Original value = £61,344 ÷ 1.728 = £35,500

Answer: £35,500

10 Perpendicular bisector

A is the point (2, 0) and B is the point (−4, −2).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $\left(\frac{2 + (-4)}{2}, \frac{0 + (-2)}{2}\right) = (-1, -1)$

Gradient of AB = $\frac{-2}{-6} = \frac{1}{3}$

Perpendicular gradient = $-1 \div \frac{1}{3} = -3$

$y = -3x + c$ through $(-1, -1)$: $-1 = -3 \times (-1) + c$, so $c = -4$
$y = -3x - 4$
Answer: $y = -3x - 4$