Day 83 **A Tier 10-a-day: Model Solutions**

1 Loci and constructions

A radio mast is at point A. Its signal reaches up to 5 km.
A radio mast at B reaches up to 7 km.
Describe the region where both signals can be received.

Each signal covers the inside of a circle
Both signals are received where the two circles overlap
Answer: Inside both circles: the overlap of a circle of radius 5 km centred on A and a circle of radius 7 km centred on B

2 Substitution

Work out the value of $3x^2 - 3x + 1$ when $x = -5$

Substitute $x = -5$: $3 \times (-5)^2 - 3 \times (-5) + 1$
$= 75 + 15 + 1$
$= 91$
Answer: 91

3 Is it a right-angled triangle?

A triangle has sides of length 24 cm, 28 cm and 10 cm.
Is the triangle right-angled?
You must show your working.

Longest side$^2 = 28^2 = 784$
Sum of the squares of the other two sides $= 10^2 + 24^2 = 100 + 576 = 676$
$784 \neq 676$, so the triangle is not right-angled
Answer: No, it is not right-angled

4 Powers and roots in standard form

Work out $(5 \times 10^4)^2$
Give your answer in standard form.

$(5 \times 10^4)^2 = 5^2 \times (10^4)^2$
$= 25 \times 10^8$
$= 2.5 \times 10^9$
Answer: 2.5×10^9

5 Independent events

The probability that Maya hits the target is 0.4.
The probability that Kai hits the target is 0.1.
The two events are independent.
Work out the probability that both Maya and Kai hit the target.

Multiply the probabilities: $0.4 \times 0.1 = 0.04$
Answer: 0.04

6 Similar shapes: area and volume

Two similar statues have heights 2 cm and 4 cm.
The surface area of the smaller one is 2 cm^2.
Work out the surface area of the larger one.

Length scale factor $= 4 \div 2 = 2$
Area scale factor $= (2)^2 = 4$
Area $= 2 \times 4 = 8$
Answer: 8 cm^2

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7 Parallel and perpendicular lines

The line L has equation $y = 5x - 7$

Find the equation of the line that is perpendicular to L and passes through the point $(0, -3)$.

The gradient of L is 5

Perpendicular gradient $= -1 \div 5 = -\frac{1}{5}$

$y = -\frac{1}{5}x + c$, so $-3 = -\frac{1}{5} \times 0 + c$

$c = -3$

$y = -\frac{1}{5}x - 3$

Answer: $y = -\frac{1}{5}x - 3$

8 HCF and LCM in index form

$A = 2^2 \times 3^2 \times 7$

$B = 2 \times 3^2 \times 7^3$

Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.

Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2 \times 3^2 \times 7$

LCM: for each prime take the higher power: $2^2 \times 3^2 \times 7^3$

Answer: HCF = $2 \times 3^2 \times 7$, LCM = $2^2 \times 3^2 \times 7^3$

9 Rationalising with conjugates

Rationalise the denominator of $\frac{28}{3 + \sqrt{2}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 - \sqrt{2}$: $\frac{28(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

Denominator: $9 - 2 = 7$

$\frac{28(3 - \sqrt{2})}{7} = 4(3 - \sqrt{2})$

$= 12 - 4\sqrt{2}$

Answer: $12 - 4\sqrt{2}$

10 Frequency density

The table gives information about 103 measurements.

A histogram is drawn using this information.

(a) Work out the missing frequency.

(b) Work out the height of the bar for the class $40 < m \leq 60$.

(a) Missing frequency $= 103 - (42 + 44 + 5) = 12$

(b) Class width $= 60 - 40 = 20$

Frequency density $= 44 \div 20 = 2.2$

Answer: (a) 12 (b) 2.2