Day 79 **A Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $2a^2 - 2ab - 3b^2$
when $a = -4$ and $b = 4$

Substitute: $2 \times (-4)^2 - 2 \times (-4) \times 4 - 3 \times 4^2$
$= 32 + 32 - 48$
$= 16$
Answer: 16

2 Converting area and volume units

Change 2000 cm^2 to m^2.

$1\ m^2 = 10\ 000\ cm^2$
$2000 \div 10\ 000 = 0.2$
Answer: 0.2 m^2

3 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 57°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
$x = 180 - 90 - 57 = 33$
Answer: x = 33°

4 Inverse proportion

7 painters take 15 days to paint a hall.
All the painters work at the same rate.
How many painters are needed to paint a hall in 7 days?

Total work = $7 \times 15 = 105$ painter-day
Number needed = $105 \div 7 = 15$
Answer: 15

5 Volume of cones and spheres

A solid hemisphere has radius 9 cm.
Work out its volume. Give your answer in terms of π.

A hemisphere is half a sphere

Volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 9^3$

$= \frac{2}{3} \times 729\ \pi$

$= 486\pi$

Answer: 486π cm^3

6 HCF and LCM in index form

$A = 2 \times 3^4 \times 5^4$

$B = 2 \times 3 \times 5^3$

Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2 \times 3 \times 5^3$

LCM: for each prime take the higher power: $2 \times 3^4 \times 5^4$

Answer: HCF = $2 \times 3 \times 5^3$, LCM = $2 \times 3^4 \times 5^4$

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7 Exact trigonometric values

PQR is a right-angled triangle with the right angle at Q.
PR = 10 cm and angle QPR = 60°.
Work out the exact length of QR.
Do not use a calculator.

QR is opposite the 60° angle and PR is the hypotenuse, so use sin

$\sin 60° = \frac{\sqrt{3}}{2}$

$QR = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$

Answer: $5\sqrt{3}$ cm

8 Inverse proportion formulae

y is inversely proportional to x.
When x = 10, y = 4.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 40.

(a) $y = \frac{k}{x}$. Substitute: $4 = \frac{k}{10}$, so $k = 4 \times 10 = 40$

$y = \frac{40}{x}$

(b) $40 = \frac{40}{x}$, so $x = 40 \div 40 = 1$

Answer: (a) $y = \frac{40}{x}$ (b) x = 1

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{4x + 3}{x + 5}$

Multiply both sides by (x + 5): $y(x + 5) = 4x + 3$
Expand the bracket: $xy + 5y = 4x + 3$
Collect the x terms on one side: $xy - 4x = -5y + 3$
Factorise: $x(y - 4) = -5y + 3$
Divide by (y − 4): $x = \frac{-5y + 3}{y - 4}$

Answer: $x = \frac{-5y + 3}{y - 4}$

10 Percentages of percentages

There are 200 passengers on a plane.
40% of the passengers are children.
40% of the children are under 5 years old.
How many passengers are children under 5?

Children = 40% of 200 = 80
Under 5 = 40% of 80 = 32
Answer: 32