Day 78 **A Tier 10-a-day: Model Solutions**

1 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 4, y = 192.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = kx^3$. Substitute x = 4 and y = 192: 192 = k × 64, so k = 3
$y = 3x^3$
(b) $y = 3 \times 5^3 = 3 \times 125 = 375$
Answer: (a) $y = 3x^3$ (b) 375

2 Negative and fractional indices

Work out the value of

(a) $216^{\frac{1}{3}}$
(b) 3^{-2}

(a) $216^{\frac{1}{3}} = \sqrt[3]{216} = 6$
(b) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Answer: (a) 6 (b) $\frac{1}{9}$

3 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 35°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 35 = 55
Answer: x = 55°

4 Double inequalities

(a) Solve $-9 < 4x + 3 < 3$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Subtract 3 from every part: $-12 < 4x < 0$
Divide every part by 4: $-3 < x < 0$
(b) The integers are −2, −1
Answer: (a) $-3 < x < 0$ (b) −2, −1

5 Percentages in context

A laptop costs £500 plus VAT at 20%.
In a sale, the price including VAT is reduced by 20%.
Work out the sale price of the laptop.

VAT = 20% of £500 = £100, so the price with VAT is £600
Reduction = 20% of £600 = £120
Sale price = £600 − £120 = £480
Answer: £480

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6 Reverse fractions

Femi reads $\frac{3}{4}$ of a book and has now read 39 pages.

How many pages does the whole book have?

$\frac{3}{4}$ of the total = 39

$\frac{1}{4}$ of the total = 39 ÷ 3 = 13

Whole = 13 × 4 = 52

Answer: 52

7 Pythagoras in three dimensions

The diagram shows a cuboid.
Work out the length of the diagonal AG. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$AG^2 = 9^2 + 11^2 + 4^2$

$= 81 + 121 + 16 = 218$

$AG = \sqrt{218} = 14.8$

Answer: 14.8 cm

8 Reading a histogram

The histogram shows information about the time of some items.
(a) Work out the number of items with a time in the class 40 to 50 minutes.
(b) Work out an estimate for the number with a time greater than 35 minutes.

(a) Frequency = frequency density × class width

1.2 × 10 = 12

(b) 35 to 40: 1.8 × 5 = 9

Class 40 to 50: 1.2 × 10 = 12

Class 50 to 80: 2 × 30 = 60

Total = 9 + 12 + 60 = 81

Answer: (a) 12 (b) 81

9 Bounds and accuracy

A cyclist travels 300 metres in 14.3 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 305 ÷ 14.25 = 21.4035

Lower bound = 295 ÷ 14.35 = 20.5575

(b) Both bounds round to 21 to 2 s.f. but not to the same value to 3 s.f.

So the speed is 21 m/s to 2 significant figures.

Answer: (a) 21.4035, 20.5575 (b) 21 m/s (2 s.f.)

10 Volume of cones and spheres

A solid hemisphere has radius 6 cm.
Work out its volume. Give your answer in terms of π.

A hemisphere is half a sphere

Volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 6^3$

$= \frac{2}{3} \times 216\ \pi$

$= 144\pi$

Answer: 144π cm^3