Day 76 **A Tier 10-a-day: Model Solutions**

1 nth term of a linear sequence

Here are the first four terms of a sequence.
17, 14, 11, 8
(a) Find an expression for the nth term.
(b) Find the first term in the sequence that is less than −100.

The terms change by −3 each time, so the nth term starts with −3n
17 − (−3) = 20, so the nth term is −3n + 20
(b) −3n + 20 < −100
−3n < −120
Divide by −3 and reverse the inequality sign: n > 40
The smallest whole number is n = 41, and the term is −103
Answer: (a) −3n + 20 (b) −103

2 Direct proportion

Here are the ingredients for 6 muffins.
Sugar: 90 g
Oats: 150 g
Flour: 180 g
(a) Work out how much sugar is needed for 30 muffins.
(b) Hannah has 324 g of oats. What is the greatest number of muffins she can make?

(a) sugar for one: 90 ÷ 6 = 15 g
For 30: 15 × 30 = 450 g
(b) oats for one: 150 ÷ 6 = 25 g
324 ÷ 25 = 12 remainder 24, so she can make 12
Answer: (a) 450 g (b) 12

3 Similar shapes: area and volume

Two similar bottles have heights 12 cm and 18 cm.
The surface area of the smaller one is 32 cm^2.
Work out the surface area of the larger one.

Length scale factor = 18 ÷ 12 = $\frac{3}{2}$

Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$

Area = $32 \times \frac{9}{4} = 72$

Answer: 72 cm^2

4 Profit and loss

Leah buys a bike for £320 and sells it for £352.
Work out Leah's percentage profit.

Profit = £352 − £320 = £32

Percentage profit = $\frac{32}{320} \times 100 = 10\%$

Answer: 10%

5 Forming and solving equations

An isosceles triangle has two equal sides of length (2x + 10) cm and (x + 20) cm.
The third side is 6 cm.
Work out the perimeter of the triangle.

The two equal sides give 2x + 10 = x + 20
x = 10
x = 10
Equal sides = 2 × 10 + 10 = 30 cm
Perimeter = 30 + 30 + 6 = 66
Answer: 66 cm

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6 Compound interest

Ravi invests £1000 in a savings account. (Calculator)
The account pays 8% compound interest per year.
Work out the interest earned in 3 years.

Multiplier = 100% + 8% = 1.08
£1000 × 1.08^3 = £1259.712
Interest = £1259.71 − £1000 = £259.71
Answer: £259.71

7 Circle theorems: isosceles triangles with radii

A, B and C are points on a circle with centre O.
Angle OAB = 36°.
(a) Work out angle AOB.
(b) Work out the size of angle x.
Give reasons for your answers.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 36°
Angle AOB = 180 − 36 − 36 = 108°
(b) The angle at the centre is twice the angle at the circumference
x = 108 ÷ 2 = 54
Answer: (a) 108° (b) x = 54°

8 Square and cube numbers

n is the smallest positive integer such that 441 × n is a cube number.
Find the value of n.

Write 441 as a product of prime factors: 441 = $3^2 \times 7^2$
For a cube number every prime must have a power that is a multiple of 3.
Extra primes needed: 3 (needs 1 more), 7 (needs 1 more)
n = 21
Answer: n = 21

9 Volume of a prism

The diagram shows a prism with a trapezium as its cross-section.
The volume of the prism is 1870 cm^3.
Work out the value of x.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times (9 + 13) \times 10 = 110$
Volume = area × length
1870 = 110 × length
length = 1870 ÷ 110 = 17
Answer: x = 17

10 Finding a rate of compound change

Chloe invests £4,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £5,445.
Work out the annual rate of interest.

Overall multiplier = £5,445 ÷ £4,500 = 1.21
Take the square root: $\sqrt{1.21}$ = 1.1
Rate = (1.1 − 1) × 100 = 10%
Answer: 10%