Day 68 **A Tier 10-a-day: Model Solutions**

1 Exchange rates

£1 = €1.3.
(a) Change £400 to euros.
(b) Change €234 to pounds.

(a) 400 × 1.3 = 520
(b) 234 ÷ 1.3 = 180
Answer: (a) €520 (b) £180

2 Algebraic formulae in context

A taxi company charges £2 plus £4 for each mile.
(a) Write a formula for the total cost, £C, after m miles.
(b) Aisha pays £26 in total. Work out the number of miles.

(a) C = 2 + 4m
(b) 2 + 4m = 26
4m = 24
m = 6
Answer: (a) C = 2 + 4m (b) 6

3 Counting

A shop sells 6 different shirts, 4 different pairs of trousers and 2 different hats.
How many different outfits, made of one shirt, one pair of trousers and one hat, are possible?

Multiply the number of choices: 6 × 4 × 2 = 48
Answer: 48

4 Exact trigonometric values

Without using a calculator, work out the exact value of
5 sin 30° + 5 sin 45°
Give your answer in its simplest form.

$\sin 30° = \frac{1}{2}$ and $\sin 45° = \frac{\sqrt{2}}{2}$

$5 \times \frac{1}{2} + 5 \times \frac{\sqrt{2}}{2} = \frac{5 + 5\sqrt{2}}{2}$

Answer: $\frac{5 + 5\sqrt{2}}{2}$

5 Box plots

The box plot shows the times taken, in seconds, by the students in Class A to run 100 m.
(a) Write down the median and the interquartile range.
For Class B the median is 60 and the interquartile range is 30.
(b) Compare the distributions of the two classes.

(a) Median = 45
Interquartile range = 65 − 25 = 40
(b) The median for Class B is higher (60 compared with 45), so it has the higher values on average
The IQR for Class B is smaller (30 compared with 40), so its values are more consistent
Answer: (a) Median 45, IQR 40 (b) Class B has the higher median; Class B has the smaller IQR so is more consistent

6 Expanding with surds

Expand and simplify $(6 + \sqrt{7})(6 - \sqrt{7})$

The middle terms cancel: $(6 + \sqrt{7})(6 - \sqrt{7}) = 36 - 7$
= 29
Answer: 29

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7 Surface area of cones and spheres

A cone has base radius 6 cm and perpendicular height 8 cm.
Work out the curved surface area of the cone.
Give your answer in terms of π.

Slant height $l = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$
Curved surface area = $\pi rl = \pi \times 6 \times 10$
$= 60\pi$

Answer: 60π cm^2

8 Circles and tangents

The point P(−5, −12) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-5)^2 + (-12)^2 = 169$

(b) Gradient of the radius OP = $\frac{-12}{-5} = \frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{12}{5} = -\frac{5}{12}$

$y = -\frac{5}{12}x + c$ through (−5, −12): $-12 = -\frac{5}{12} \times (-5) + c$, so $c = -\frac{169}{12}$

$y = -\frac{5}{12}x - \frac{169}{12}$

Answer: (a) $x^2 + y^2 = 169$ (b) $y = -\frac{5}{12}x - \frac{169}{12}$

9 Perpendicular bisector

A is the point (−1, 6) and B is the point (7, 0).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{(-1) + 7}{2}, \frac{6 + 0}{2}) = (3, 3)$

Gradient of AB = $\frac{-6}{8} = -\frac{3}{4}$

Perpendicular gradient = $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

$y = \frac{4}{3}x + c$ through (3, 3): $3 = \frac{4}{3} \times 3 + c$, so $c = -1$

$y = \frac{4}{3}x - 1$

Answer: $y = \frac{4}{3}x - 1$

10 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.
The outer radius is 9 cm and the inner radius is 6 cm.
The pipe is 50 cm long.
Work out the volume of metal used. Give your answer to 3 significant figures. (Calculator)

Area of the ring = $\pi \times 9^2 - \pi \times 6^2 = \pi(81 - 36) = 45\pi$
Volume = area × length = $45\pi \times 50$
$= 7070$

Answer: 7070 cm^3