Day 66 **A Tier 10-a-day: Model Solutions**

1 Bearings and distances

A ship leaves a port and travels 15 km on a bearing of 215°. (Calculator)
It then travels 8 km on a bearing of 125°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle
(a) Distance^2 = $15^2 + 8^2$ = 289, so distance = 17 km
(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{8}{15})$ = 28.1°
Bearing = 215° − 28.1° = 186.9°
Bearing = 187°
Answer: (a) 17 km (b) 187°

2 Inequalities

Solve $-3x + 8 > 23$

Subtract 8: $-3x > 15$
Divide by -3. Dividing by a negative number reverses the inequality sign
$x < -5$
Answer: $x < -5$

3 Independent events

The probability that Ella wins a game is 0.6.
The probability that Owen wins a game is 0.7.
The two events are independent.
Work out the probability that exactly one of them will win a game.

P(only Ella) = 0.6 × 0.3 = 0.18
P(only Owen) = 0.4 × 0.7 = 0.28
Add: 0.18 + 0.28 = 0.46
Answer: 0.46

4 LCM in context

Aisha runs around a track in 70 seconds.
Sara runs around the same track in 40 seconds.
They start together at the start line and keep running.
How many laps will each have run when they are next together at the start line?

Find the LCM of 70 and 40: LCM = 280 seconds.
Aisha: 280 ÷ 70 = 4 laps
Sara: 280 ÷ 40 = 7 laps
Answer: Aisha 4 laps, Sara 7 laps

5 Converting area and volume units

Change 4.5 m^2 to cm^2.

1 m = 100 cm, so 1 m^2 = 100 × 100 = 10 000 cm^2
4.5 × 10 000 = 45000
Answer: 45000 cm^2

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6 Interior angles of polygons

The size of each angle of a quadrilateral is shown as an expression:
$(x + 25)°$, $(2x + 5)°$, $(3x + 40)°$, $(x + 52)°$
Work out the value of x.

Sum of angles = $(4 - 2) \times 180 = 360°$
$x + 25 + 2x + 5 + 3x + 40 + x + 52 = 360$
$7x + 122 = 360$
$7x = 238$
$x = 34$
Answer: x = 34

7 Algebraic proof

Prove that $(n + 5)^2 - (n - 5)^2$ is a multiple of 4 for any integer n.

$(n + 5)^2 = n^2 + 10n + 25$
$(n - 5)^2 = n^2 - 10n + 25$
Difference = $20n = 4(5n)$
This is 4 × an integer, so it is a multiple of 4
Answer: Proved

8 Reverse compound change

The value of an investment grows by 10% each year. (Calculator)
After 3 years it is worth £47,916.
Work out its value at the start.

Yearly multiplier = 1.1
Overall multiplier = $1.1^3 = 1.331$
Original value = £47,916 ÷ 1.331 = £36,000
Answer: £36,000

9 Transformations of graphs

The graph of y = f(x) has a maximum point at $(-3, 5)$.
Write down the coordinates of the turning point of the graph of
(a) y = 3f(x)
(b) y = f(−x)

(a) Multiplying f(x) by 3 stretches the graph vertically by a factor of 3, so the y-coordinate is multiplied by 3
The maximum point is $(-3, 15)$
(b) The graph is reflected in the y-axis, so the x-coordinate changes sign
The maximum point is (3, 5)
Answer: (a) $(-3, 15)$ (b) (3, 5)

10 Rationalising with conjugates

Rationalise the denominator of $\frac{31}{6 + \sqrt{5}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $6 - \sqrt{5}$: $\frac{31(6 - \sqrt{5})}{(6 + \sqrt{5})(6 - \sqrt{5})}$

Denominator: $36 - 5 = 31$
$\frac{31(6 - \sqrt{5})}{31} = 1(6 - \sqrt{5})$
$= 6 - \sqrt{5}$
Answer: $6 - \sqrt{5}$