Day 63 **A Tier 10-a-day: Model Solutions**

1 Algebraic formulae in context

A taxi company charges £4 plus £2 for each mile.
(a) Write a formula for the total cost, £C, after m miles.
(b) Maria pays £32 in total. Work out the number of miles.

(a) C = 4 + 2m
(b) 4 + 2m = 32
2m = 28
m = 14
Answer: (a) C = 4 + 2m (b) 14

2 Pythagoras' theorem

Work out the length of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$x^2 = 3^2 + 9^2$
$x^2 = 9 + 81 = 90$
$x = \sqrt{90} = 9.49$
Answer: x = 9.49 mm

3 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 80 frogs, marks each one and returns them.
Later she catches a second sample of 60 frogs. 20 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{20}{60}$

This is the same as the proportion in the whole population: $\frac{80}{N}$

$\frac{80}{N} = \frac{20}{60}$, so N = 80 × 60 ÷ 20 = 240
Answer: 240

4 Double inequalities

(a) Solve $-18 \leq 5x + 2 \leq -3$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Subtract 2 from every part: $-20 \leq 5x \leq -5$
Divide every part by 5: $-4 \leq x \leq -1$
(b) The integers are −4, −3, −2, −1
Answer: (a) $-4 \leq x \leq -1$ (b) −4, −3, −2, −1

5 Compound interest

Ben invests £1000 in a savings account. (Calculator)
The account pays 6% compound interest per year.
Work out the amount in the account after 4 years.

Multiplier = 100% + 6% = 1.06
£1000 × 1.06^4 = £1262.477
= £1262.48 to the nearest penny
Answer: £1262.48

6 Simplifying surds

(a) Simplify $\sqrt{147}$
(b) Simplify $\sqrt{147} + \sqrt{48}$

(a) $\sqrt{147} = \sqrt{49} \times \sqrt{3} = 7\sqrt{3}$
(b) $\sqrt{48} = 4\sqrt{3}$, so $7\sqrt{3} + 4\sqrt{3} = 11\sqrt{3}$
Answer: (a) $7\sqrt{3}$ (b) $11\sqrt{3}$

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7 Inverse functions

f(x) = 6x + 2

(a) Find $f^{-1}(x)$

(b) Work out $f^{-1}(-2)$

Let y = 6x + 2

Rearrange to make x the subject: $6x = y - 2$, so $x = \frac{y - 2}{6}$

Swap y for x: $f^{-1}(x) = \frac{x - 2}{6}$

(b) $f^{-1}(-2) = -\frac{2}{3}$

Answer: (a) $f^{-1}(x) = \frac{x - 2}{6}$ (b) $-\frac{2}{3}$

8 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{24^2 + 7^2}$ = 25 cm

Two triangular ends: $2 \times \frac{1}{2} \times 24 \times 7 = 168$

Three rectangles: (24 + 7 + 25) × 13 = 728

Total = 168 + 728 = 896

Answer: 896 cm^2

9 Reverse compound change

The value of a painting increases by 10% each year. (Calculator)
After 2 years it is worth £38,115.
Work out its value at the start.

Yearly multiplier = 1.1

Overall multiplier = 1.1^2 = 1.21

Original value = £38,115 ÷ 1.21 = £31,500

Answer: £31,500

10 Surface area of cones and spheres

A solid cone has base radius 12 cm and perpendicular height 9 cm.
Work out the total surface area of the cone.
Give your answer in terms of π.

Slant height l = $\sqrt{12^2 + 9^2}$ = 15

Curved surface = πrl = 180π

Base = πr^2 = 144π

Total = 180π + 144π = 324π

Answer: 324π cm^2