Day 62 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 98°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
x = 360 − 90 − 90 − 98 = 82
Answer: x = 82°

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.
0.32 94% $\frac{3}{5}$ $\frac{1}{4}$

Change each to a decimal: 0.32 = 0.32, 94% = 0.94, $\frac{3}{5}$ = 0.6, $\frac{1}{4}$ = 0.25
Order the decimals: 0.25 < 0.32 < 0.6 < 0.94
Answer: $\frac{1}{4}$, 0.32, $\frac{3}{5}$, 94%

3 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the opposite (20) and the adjacent (11), so use tan
tan x = $\frac{20}{11}$
x = $\tan^{-1}(\frac{20}{11})$
x = 61.2°
Answer: x = 61.2°

4 Rearranging formulae

Make x the subject of $y = \frac{5x - 4}{4}$

Multiply both sides by 4: 4y = 5x − 4
Add 4 to both sides: 5x = 4y + 4
Divide both sides by 5
x = $\frac{4y + 4}{5}$
Answer: x = $\frac{4y + 4}{5}$

5 Income tax

No tax is paid on the first £10,000 of income. Tax of 25% is paid on the next £25,000. Tax of 45% is paid on all income above £35,000. (Calculator)
Jack earns £53,000 a year.
Work out the income tax paid.

Next £25,000 at 25%: £25,000 × 0.25 = £6,250
Above £35,000: £53,000 − £35,000 = £18,000, taxed at 45%: £18,000 × 0.45 = £8,100
Total tax = £6,250 + £8,100 = £14,350
Answer: £14,350

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6 Inverse proportion formulae

y is inversely proportional to x^2.
When x = 4, y = 10.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = \frac{k}{x^2}$. Substitute: $10 = \frac{k}{16}$, so k = 10 × 16 = 160

$y = \frac{160}{x^2}$

(b) $y = \frac{160}{25} = 6.4$

Answer: (a) $y = \frac{160}{x^2}$ (b) 6.4

7 Difference of two squares

(a) Factorise $4y^2 - 9z^2$
(b) Use the same idea to work out $167^2 - 33^2$ without a calculator.

(a) $4y^2 - 9z^2 = (2y)^2 - (3z)^2$
$(2y - 3z)(2y + 3z)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $167^2 - 33^2 = (167 + 33)(167 - 33)$
$= 200 \times 134 = 26800$
Answer: (a) (2y − 3z)(2y + 3z) (b) 26800

8 The cosine rule: finding a side

Work out the value of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$x^2 = 5^2 + 19^2 - 2 \times 5 \times 19 \times \cos 74°$
$x^2 = 25 + 361 - 190 \cos 74°$
$x^2 = 333.63$
x = 18.3
Answer: x = 18.3 m

9 Distance-time graphs

The graph shows the distance travelled by a cyclist.
(a) For how many minutes did the cyclist stop?
(b) Work out the speed during the first part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 30 to 40 minutes, so the stop is 40 − 30 = 10 minutes
(b) First part: distance 10 km in 30 minutes

Speed = distance ÷ time = $10 \div \frac{30}{60} = 20$ km/h

Answer: (a) 10 minutes (b) 20 km/h

10 Equations with algebraic fractions

Solve $\frac{4}{(x + 3)} + \frac{1}{(x - 3)} = 1$

Multiply every term by (x + 3)(x − 3)
$4(x - 3) + 1(x + 3) = (x + 3)(x - 3)$
$5x - 9 = x^2 - 9$
$x^2 - 5x = 0$
$x(x - 5) = 0$
x = 0 or x = 5
Answer: x = 0 or x = 5