Day 60 **A Tier 10-a-day: Model Solutions**

1 Rearranging formulae

Make p the subject of $q = 9(p - 5)$

Divide both sides by 9: $\frac{q}{9} = p - 5$

Add 5 to both sides

$p = \frac{q}{9} + 5$

Answer: $p = \frac{q}{9} + 5$

2 Isosceles triangles and exterior angles

LMN is an isosceles triangle with LM = LN.
MNK is a straight line.
Angle LNK = 126°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle LNM = 180 − 126 = 54°
Base angles are equal, so angle LMN = 54°
x = 180 − 54 − 54 = 72
Answer: x = 72°

3 Sequences

Here are the first four terms of a geometric sequence.
2, 8, 32, 128
(a) Write down the next two terms.
(b) Work out the 6th term.

Divide each term by the one before: 8 ÷ 2 = 4, so each term is multiplied by 4
(a) 128 × 4 = 512 and 512 × 4 = 2048
(b) Keep multiplying: 6th term = $2 \times 4^5 = 2048$
Answer: (a) 512, 2048 (b) 2048

4 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 80 frogs, marks each one and returns them.
Later she catches a second sample of 60 frogs. 4 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{4}{60}$

This is the same as the proportion in the whole population: $\frac{80}{N}$

$\frac{80}{N} = \frac{4}{60}$, so N = 80 × 60 ÷ 4 = 1200
Answer: 1200

5 Index equations

Solve $8^x = 2$

Write both sides as powers of 2: $8 = 2^3$, so $8^x = 2^{3x}$
$2 = 2^1$, so 3x = 1
$x = \frac{1}{3}$

Answer: $x = \frac{1}{3}$

Day 60 **A Tier 10-a-day: Model Solutions**

6 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 46°.
Work out the size of angle x (angle OAB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
Angle AOB = 360 − 90 − 90 − 46 = 134°
OA = OB (radii), so triangle OAB is isosceles
x = (180 − 134) ÷ 2 = 23
Answer: x = 23°

7 Pressure

A block exerts a force of 300 N on the floor.
The pressure on the floor is 120 N/m^2.
Work out the area of the block in contact with the floor.

Area = force ÷ pressure
= 300 ÷ 120 = 2.5 m^2
Answer: 2.5 m^2

8 Recurring decimals, show that

(a) Show that $0.\dot{9}\dot{6} = \frac{32}{33}$

(b) Hence write $4.\dot{9}\dot{6}$ as a mixed number in its simplest form.

(a) Let x = 0.969696..., so 100x = 96.9696...
100x − x = 96, so 99x = 96 and $x = \frac{96}{99} = \frac{32}{33}$

(b) $4.\dot{9}\dot{6} = 4 + \frac{32}{33} = 4\frac{32}{33}$

Answer: (b) $4\frac{32}{33}$

9 Transformations of graphs

The graph of y = f(x) has a maximum point at (0, 2).
Write down the coordinates of the turning point of the graph of
(a) y = f(2x)
(b) y = f(x) + 3

(a) Replacing x by 2x squeezes the graph horizontally by a factor of 2, so the x-coordinate is halved
The maximum point is (0, 2)
(b) Adding 3 to f(x) moves the graph 3 units up
The maximum point is (0, 5)
Answer: (a) (0, 2) (b) (0, 5)

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 56° and angle ACD = 62°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 62°
In triangle ABE: angle BAE = 56° and angle ABE = 62°
x = 180 − 56 − 62 = 62
Answer: x = 62°