Day 6 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: tangents

PA is a tangent to a circle with centre O. A is on the circle.
The radius is 12 m and OP = 13 m.
Work out the length of PA.
The diagram is not drawn accurately.

The tangent is perpendicular to the radius, so angle OAP = 90°
$PA^2 = 13^2 - 12^2 = 169 - 144 = 25$
$PA = \sqrt{25} = 5$
Answer: 5 m

2 Exchange rates

The exchange rate is £1 = ¥190. (Calculator)
Nadia changes £1,150 into Japanese yen.
How much does Nadia get in Japanese yen?

£1,150 × 190 = ¥218,500
Answer: ¥218,500

3 Equations with brackets

Solve $-3(x - 1) = -2(x + 2) + 2$

Expand the brackets: $-3x + 3 = -2x - 4 + 2$
Simplify: $-3x + 3 = -2x - 2$
Collect x terms on one side and numbers on the other: $-x = -5$
$x = 5$
Answer: x = 5

4 Converting area and volume units

A swimming pool holds 1.2 m^3 of water.
How many litres is this?

1 m^3 = 1000 litres
1.2 × 1000 = 1200
Answer: 1200 litres

5 Quadratic graphs

The curve C has equation $y = x^2 - 8x + 7$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set y = 0: $x^2 - 8x + 7 = 0$
$(x - 1)(x - 7) = 0$
x = 1 or x = 7, so the graph crosses the x-axis at (1, 0) and (7, 0)
(b) The turning point is halfway between the roots: $x = (1 + 7) \div 2 = 4$
$y = -9$, so the minimum point is (4, −9)
Answer: (a) (1, 0) and (7, 0) (b) (4, −9)

6 Bounds

A rectangle has length 10 mm and width 22 mm.
Each measurement is correct to the nearest mm.
Work out the upper bound for the perimeter of the rectangle.

Upper bound of length = 10.5 mm, upper bound of width = 22.5 mm
Perimeter = 2 × (10.5 + 22.5) = 66 mm
Answer: 66 mm

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7 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)

Scale factor = AB ÷ AD = 11 ÷ 3 = $\frac{11}{3}$

x = 6 × $\frac{11}{3}$ = 22

Answer: x = 22

8 Venn diagrams from information

There are 26 students in a class.
7 students do football and 14 students do hockey.
9 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do football only.

(a) Students doing at least one = 26 − 9 = 17
Both = 7 + 14 − 17 = 4
(b) football only = 7 − 4 = 3

Probability = $\frac{3}{26}$

Answer: (a) 4 (b) $\frac{3}{26}$

9 Frequency density

The table gives information about 119 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $25 < m \leq 40$.

(a) Missing frequency = 119 − (33 + 52 + 7) = 27
(b) Class width = 40 − 25 = 15
Frequency density = 33 ÷ 15 = 2.2
Answer: (a) 27 (b) 2.2

10 Rationalising with conjugates

Rationalise the denominator of $\frac{10}{4-\sqrt{6}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $4 + \sqrt{6}$: $\frac{10(4+\sqrt{6})}{(4-\sqrt{6})(4+\sqrt{6})}$

Denominator: 16 − 6 = 10

$\frac{10(4+\sqrt{6})}{10} = 1(4+\sqrt{6})$

$= 4 + \sqrt{6}$

Answer: $4 + \sqrt{6}$