Day 57 **A Tier 10-a-day: Model Solutions**

1 Estimation

By rounding each number to 1 significant figure, estimate the value of

$$\frac{27.2 + 2.8}{0.54}$$

Round each number to 1 significant figure: 30, 3, 0.5

$$\frac{30 + 3}{0.5} = 66$$

Answer: 66

2 Stratified sampling

A company has 220 employees in Sales, 100 in Admin and 180 in Production.
A stratified sample of 25 employees is to be taken.
Work out how many employees should be chosen from Admin.

Total = 220 + 100 + 180 = 500

Number from Admin = $\frac{100}{500} \times 25$

= 5

Answer: 5

3 Counting

A cafe menu has 5 starters, 3 main courses and 5 desserts.
(a) How many different three-course meals are possible?
(b) A customer chooses a main course and either a starter or a dessert (not both). How many different meals are possible?

(a) 5 × 3 × 5 = 75
(b) 3 choices of main course × (5 + 5) choices of starter or dessert = 3 × 10 = 30
Answer: (a) 75 (b) 30

4 Surface area of a cuboid

A cuboid has length 6 cm and width 3 cm.
Its total surface area is 144 cm^2.
Work out the height of the cuboid.

2 × (6 × 3 + 6 h + 3 h) = 144
18 + 9h = 72
9h = 54
h = 54 ÷ 9 = 6
Answer: 6 cm

5 Estimating the mean from grouped data

The table shows the distances, in km, travelled by 30 lorries.
(a) Write down the class that contains the median.
(b) Work out an estimate for the mean. Give your answer to 1 decimal place.

(a) The median is in position (30 + 1) ÷ 2 = 15.5. Cumulative frequencies: 3, 5, 23, 30
The median is in the class $20 < d \leq 30$
(b) Midpoints: 5, 15, 25, 35
5 × 3 + 15 × 2 + 25 × 18 + 35 × 7 = 740
Estimate = 740 ÷ 30 = 24.7
Answer: (a) $20 < d \leq 30$ (b) 24.7

Day 57 **A Tier 10-a-day: Model Solutions**

6 Completing the square

(a) Write $x^2 - 2x - 3$ in the form $(x + a)^2 + b$

(b) Write down the coordinates of the turning point of the curve $y = x^2 - 2x - 3$

(a) Half of -2 is -1, so $(x - 1)^2$

$(x - 1)^2 = x^2 - 2x + 1$, so subtract 1 to get back to -3

So $x^2 - 2x - 3 = (x - 1)^2 - 4$

(b) The turning point is where the bracket is 0, so $x = 1$ and $y = -4$

Answer: (a) $(x - 1)^2 - 4$ (b) (1, −4)

7 Area of shaded regions with circles

A circle of radius 5 cm fits exactly inside a square.
The circle touches all four sides of the square. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Side of square = 2 × 5 = 10 cm

Square: 10 × 10 = 100

Circle: $\pi \times 5^2 = 25\pi$

Shaded area = $100 - 25\pi = 21.5$

Answer: 21.5 cm^2

8 Cumulative frequency graphs

The cumulative frequency graph shows information about the mass of 40 parcels.
Use the graph to find an estimate for
(a) the median,
(b) the number of parcels with a mass greater than 100 kg.

(a) Median at cumulative frequency $40 \div 2 = 20$

Read across from 20 to the graph, then down to the axis

Median = 64 kg

(b) At 100 kg the cumulative frequency is 29

$40 - 29 = 11$

Answer: (a) 64 kg (b) 11

9 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{5x - 2}{x - 5}$

Multiply both sides by $(x - 5)$: $y(x - 5) = 5x - 2$

Expand the bracket: $xy - 5y = 5x - 2$

Collect the x terms on one side: $xy - 5x = 5y - 2$

Factorise: $x(y - 5) = 5y - 2$

Divide by $(y - 5)$: $x = \frac{5y - 2}{y - 5}$

Answer: $x = \frac{5y - 2}{y - 5}$

10 Converting units of area and volume

Convert 90 mm^2 into cm^2.

1 cm = 10 mm, so 1 cm^2 = 10 × 10 = 100 mm^2

$90 \div 100 = 0.9$

Answer: 0.9 cm^2