Day 55 **A Tier 10-a-day: Model Solutions**

1 Standard form notation

(a) Write 0.0089 in standard form.
(b) Write 3.4×10^4 as an ordinary number.

(a) Move the decimal point 3 places to get 8.9: 8.9×10^{-3}
(b) Move the decimal point 4 places to the right: 34,000

Answer: (a) 8.9×10^{-3} (b) 34,000

2 Density

A piece of plastic has a volume of 40 cm^3.
The density of the material is 7.8 g/cm^3.
Work out the mass of the piece of plastic.

Mass = density × volume
= 7.8 × 40 = 312 g
Answer: 312 g

3 Calculator and significant figures

Work out $4.4 \times (1 + 2.6 \div 100)^3$ (Calculator)
Give your answer correct to 3 significant figures.

1 + 2.6 ÷ 100 = 1.026
= 4.75220...
= 4.75 to 3 significant figures
Answer: 4.75

4 Area of shaded regions with circles

2 identical circles of radius 5 cm fit exactly inside a rectangle.
The circles touch each other and the sides of the rectangle. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Rectangle: $20 \times 10 = 200$ cm^2
2 circles: $2 \times \pi \times 5^2 = 50\pi$
Shaded area = $200 - 50\pi = 42.9$
Answer: 42.9 cm^2

5 Forming and solving equations

The three angles of a triangle are x°, 4x + 16° and 2x − 46°.
Work out the value of x.

Angles in a triangle add up to 180°
x + 4x + 16 + 2x − 46 = 180
7x − 30 = 180
7x = 210
x = 30
Answer: x = 30

6 Average speed

A cyclist travels for 3 hours at 25 km/h,
then for 2 hours 30 minutes at 80 km/h.
Work out the average speed for the whole journey.

Distance 1 = 25 × 3 = 75 km
Distance 2 = 80 × 2.5 = 200 km
Total distance = 275 km and total time = 5.5 hours
Average speed = 275 ÷ 5.5 = 50 km/h
Answer: 50 km/h

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7 Bearings with the cosine rule

Ship A sails 25 km from a port on a bearing of 075°.
Ship B sails 10 km from the same port on a bearing of 340°. (Calculator)
Work out the distance between the two ships.
Give your answer to 3 significant figures.

The angle between the two paths = $360° - |75° - 340°| = 95°$
$d^2 = 25^2 + 10^2 - 2 \times 25 \times 10 \times \cos 95°$
$d^2 = 768.58$
$d = 27.7$
Answer: 27.7 km

8 Circles and tangents

The point P(5, −12) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 5^2 + (-12)^2 = 169$

(b) Gradient of the radius OP = $\frac{-12}{5}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{12}{5}) = \frac{5}{12}$

$y = \frac{5}{12}x + c$ through (5, −12): $-12 = \frac{5}{12} \times 5 + c$, so $c = -\frac{169}{12}$

$y = \frac{5}{12}x - \frac{169}{12}$

Answer: (a) $x^2 + y^2 = 169$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

9 Reverse percentages

The price of a train ticket increases by 15%.
The new price is £575.
Work out the price before the increase.

115% represents £575
1% = £575 ÷ 115 = £5
100% = £5 × 100 = £500.00
Answer: £500.00

10 Equations with algebraic fractions

Solve $\frac{6}{(x+4)} + \frac{2}{(x-4)} = 1$

Multiply every term by $(x + 4)(x - 4)$
$6(x - 4) + 2(x + 4) = (x + 4)(x - 4)$
$8x - 16 = x^2 - 16$
$x^2 - 8x = 0$
$x(x - 8) = 0$
$x = 0$ or $x = 8$
Answer: $x = 0$ or $x = 8$