Day 54 **A Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{4x + 1}{3} = \frac{5x + 3}{4}$

Multiply both sides by 12
$4(4x + 1) = 3(5x + 3)$
$16x + 4 = 15x + 9$
$x = 5$
$x = 5$
Answer: x = 5

2 Time

(a) A coach leaves Leeds at 4:55 pm. The journey takes 3 hours 28 minutes. Work out the time it ends.
(b) Change 1.1 hours into hours and minutes.

(a) 4:55 pm + 3 hours = 7:55 pm, then + 28 minutes = 8:23 pm
(b) 0.1 hours = 0.1 × 60 = 6 minutes, so 1.1 hours = 1 hours 6 minutes
Answer: (a) 8:23 pm (b) 1 hours 6 minutes

3 Circle area and circumference

The area of a circle is 121π cm^2.
(a) Work out the radius of the circle.
(b) Work out the circumference. Give your answer in terms of π.

(a) $\pi r^2 = 121\pi$, so $r^2 = 121$
$r = 11$
(b) Circumference $= 2\pi r = 2 \times \pi \times 11 = 22\pi$
Answer: (a) 11 cm (b) 22π cm

4 Laws of indices

Simplify fully
(a) $\frac{9x^2y^5}{4x^5y^3}$
(b) $(3x^3y^3)^3$

(a) Divide the numbers: $9 \div 4 = \frac{9}{4}$
Subtract the powers: $x^2 \div x^5 = x^{-3}$ and $y^5 \div y^3 = y^2$
$= \frac{9y^2}{4x^3}$
(b) Raise every part to the power 3: $3^3 = 27$, $(x^3)^3 = x^9$, $(y^3)^3 = y^9$
$= 27x^9y^9$
Answer: (a) $\frac{9y^2}{4x^3}$ (b) $27x^9y^9$

5 Midpoints and length of a line segment

M is the midpoint of AB.
A is the point $(-3, 5)$ and M is the point $(4, 6)$.
Work out the coordinates of B.

From A to M: x changes by 7 and y changes by 1
Do the same again from M to get B
B = (4 + 7, 6 + 1) = (11, 7)
Answer: (11, 7)

6 Sharing in a ratio

Femi and Maya share some money in the ratio 5 : 6.
Femi gets £50.
How much does Maya get?

5 parts = £50, so 1 part = £50 ÷ 5 = £10
Maya gets 6 parts: 6 × £10 = £60
Answer: £60

7 Growth and decay

A colony has 9,500 and grows by 12% per day. (Calculator)
Work out the number of bacteria after 4 days.
Give your answer to the nearest whole number.

Multiplier = 1.12
$9{,}500 \times 1.12^4 = 14{,}948.43$
= 14,948 to the nearest whole number
Answer: 14,948

8 Forming quadratic equations

A rectangle has width x cm and length (x + 6) cm.
The area of the rectangle is 27 cm^2.
(a) Show that $x^2 + 6x - 27 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 6) = 27$
$x^2 + 6x = 27$
$x^2 + 6x - 27 = 0$
(b) Factorise: $(x - 3)(x + 9) = 0$
$x = 3$ or $x = -9$
A length cannot be negative, so $x = 3$
Answer: (a) Shown (b) x = 3

9 Probability without replacement

A bag contains 5 red and 3 blue counters.
Two counters are taken at random without replacement.
Work out the probability that the two counters are the same colour.

P(red, red) = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

P(blue, blue) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

Add: $\frac{20}{56} + \frac{6}{56} = \frac{13}{28}$

Answer: $\frac{13}{28}$

10 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$\cos x = \frac{18^2 + 20^2 - 11^2}{2 \times 18 \times 20}$

$\cos x = \frac{603}{720}$

$x = \cos^{-1}(0.8375)$
$x = 33.1°$
Answer: x = 33.1°