Day 51 **A Tier 10-a-day: Model Solutions**

1 Area of parallelograms and kites

The diagram shows a kite.
The diagonals are 13 cm and 9 cm and cross at right angles.
Work out the area of the kite.
The diagram is not drawn accurately.

Area = $\frac{1}{2}$ × (diagonal 1) × (diagonal 2)

= $\frac{1}{2}$ × 13 × 9

= 58.5

Answer: 58.5 cm^2

2 Capture-recapture

A scientist wants to estimate the number of fish in a lake.
She catches 60 fish, marks each one and returns them.
Later she catches a second sample of 40 fish. 4 of these are tagged.
Work out an estimate for the total number of fish in a lake.

Proportion tagged in the second sample = $\frac{4}{40}$

This is the same as the proportion in the whole population: $\frac{60}{N}$

$\frac{60}{N} = \frac{4}{40}$, so N = 60 × 40 ÷ 4 = 600

Answer: 600

3 Income tax

No tax is paid on the first £11,000 of income. Tax of 25% is paid on the next £35,000. Tax of 40% is paid on all income above £46,000. (Calculator)
Femi earns £54,500 a year.
Work out the income tax paid.

Next £35,000 at 25%: £35,000 × 0.25 = £8,750
Above £46,000: £54,500 − £46,000 = £8,500, taxed at 40%: £8,500 × 0.4 = £3,400
Total tax = £8,750 + £3,400 = £12,150
Answer: £12,150

4 Simultaneous equations

Solve the simultaneous equations
−x + 2y = −1
4x + y = 22

Multiply (2) by 2 to make the y coefficients match
−x + 2y = −1 (1)
8x + 2y = 44 (2)
The y terms have the same sign, so subtract equation (1) from equation (2)
9x = 45, so x = 5
Substitute x = 5 into equation (1) to find y = 2
Answer: x = 5, y = 2

5 Percentage change

The population of a village changes from 500 to 350.
Work out the percentage decrease.

Decrease = 500 − 350 = 150

Percentage decrease = $\frac{150}{500}$ × 100 = 30%

Answer: 30%

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6 Equations with algebraic fractions

Solve $\frac{1}{(x-5)} + \frac{6}{(x+1)} = 1$

Multiply every term by $(x - 5)(x + 1)$

$1(x + 1) + 6(x - 5) = (x - 5)(x + 1)$

$7x - 29 = x^2 - 4x - 5$

$x^2 - 11x + 24 = 0$

$(x - 3)(x - 8) = 0$

$x = 3$ or $x = 8$

Answer: $x = 3$ or $x = 8$

7 Volume of a pyramid

The diagram shows a pyramid with a square base of side 9 cm.
The perpendicular height of the pyramid is 9 cm.
Work out the volume of the pyramid.
The diagram is not drawn accurately.

Volume = $\frac{1}{3}$ × base area × height

= $\frac{1}{3} \times 9 \times 9 \times 9$

= 243

Answer: 243 cm^3

8 Cumulative frequency graphs

The cumulative frequency graph shows information about the age of 80 visitors.
Use the graph to find an estimate for
(a) the median,
(b) the number of visitors with a age greater than 30 years.

(a) Median at cumulative frequency $80 \div 2 = 40$
Read across from 40 to the graph, then down to the axis
Median = 38 years
(b) At 30 years the cumulative frequency is 32
$80 - 32 = 48$
Answer: (a) 38 years (b) 48

9 Bounds

A triangle has base 13 cm and perpendicular height 8 cm.
Each measurement is correct to the nearest cm.
Work out the lower bound for the area of the triangle.

Lower bounds: base = 12.5 cm, height = 7.5 cm

Area = ½ × 12.5 × 7.5 = 46.875 cm^2

Answer: 46.875 cm^2

10 Quadratic inequalities

Solve $x^2 - 13x + 30 \leq 0$

Solve $x^2 - 13x + 30 = 0$: $(x - 3)(x - 10) = 0$
$x = 3$ or $x = 10$
Sketch the U-shaped curve $y = x^2 - 13x + 30$, crossing the x-axis at 3 and 10
The curve is below the x-axis between the roots
$3 \leq x \leq 10$
Answer: $3 \leq x \leq 10$