Day 50 **A Tier 10-a-day: Model Solutions**

1 Tree diagrams

The tree diagram shows probabilities for two events.
(a) Work out the values of x and y.
(b) Work out the probability that a part is rejected at the final check.

(a) Probabilities on pairs of branches add up to 1.
x = 1 − 0.3 = 0.7 and y = 1 − 0.4 = 0.6
(b) P(rejected) = 0.3 × 0.4 + 0.7 × 0.3
= 0.12 + 0.21 = 0.33
Answer: (a) x = 0.7, y = 0.6 (b) 0.33

2 Interior angles of polygons

The size of each angle of a quadrilateral is shown as an expression:
(2x − 5)°, (3x − 25)°, (2x + 35)°, (3x + 25)°
Work out the value of x.

Sum of angles = (4 − 2) × 180 = 360°
2x − 5 + 3x − 25 + 2x + 35 + 3x + 25 = 360
10x + 30 = 360
10x = 330
x = 33
Answer: x = 33

3 Equation of a line from a graph

Work out the equation of the straight line.
Give your answer in the form y = mx + c

The line crosses the y-axis at (0, 3), so c = 3

Gradient = $\frac{\text{change in y}}{\text{change in x}}$: from (0, 3) to (−4, 5), y changes by 2 and x changes by −4

m = $-\frac{1}{2}$

y = $-\frac{1}{2}$x + 3

Answer: y = $-\frac{1}{2}$x + 3

4 Recurring decimals, show that

(a) Show that $0.\dot{9}\dot{5} = \frac{95}{99}$

(b) Hence write $1.\dot{9}\dot{5}$ as a mixed number in its simplest form.

(a) Let x = 0.959595..., so 100x = 95.9595...

100x − x = 95, so 99x = 95 and x = $\frac{95}{99}$

(b) $1.\dot{9}\dot{5} = 1 + \frac{95}{99} = 1\frac{95}{99}$

Answer: (b) $1\frac{95}{99}$

5 Is it a right-angled triangle?

A triangle has sides of length 4 m, 6 m and 3 m.
Is the triangle right-angled?
You must show your working.

Longest side^2 = 6^2 = 36
Sum of the squares of the other two sides = $3^2 + 4^2$ = 9 + 16 = 25
36 ≠ 25, so the triangle is not right-angled
Answer: No, it is not right-angled

6 Recurring decimals (mixed)

Write $0.4\dot{2}$ as a fraction in its simplest form.

Let x = 0.422...
Multiply by 10: 10x = 4.22...
Multiply by 100: 100x = 42.22...
Subtract: 90x = 42 − 4 = 38
$x = \frac{38}{90} = \frac{19}{45}$

Answer: $\frac{19}{45}$

7 Quadratic sequences

Here are the first five terms of a quadratic sequence.
−3, −13, −27, −45, −67
(a) Find an expression for the nth term.
(b) Work out the 8th term.

First differences: −10, −14, −18
Second differences: −4, −4, so a = −4 ÷ 2 = −2
Subtract $-2n^2$ from each term (−2, −8, −18, −32): −1, −5, −9, −13
This is linear: −4n + 3
nth term = $-2n^2 - 4n + 3$
(b) Substitute n = 8: $-2 \times 8^2 - 4 \times 8 + 3 = -157$

Answer: (a) $-2n^2 - 4n + 3$ (b) −157

8 Error intervals

m = 900 is rounded to 1 significant figure.
Write down the error interval for m.

The rounding unit is 100, so the value could be up to half of this, 50, either side.
Lower bound = 900 − 50 = 850, upper bound = 900 + 50 = 950
Answer: $850 \leq m < 950$

9 Pressure

A table exerts a force of 80 N on the floor.
The area in contact with the floor is 4 m^2.
Work out the pressure on the floor in N/m^2.

Pressure = force ÷ area
= 80 ÷ 4 = 20
Answer: 20 N/m^2

10 Circles and tangents

The point P(8, −6) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 8^2 + (-6)^2 = 100$

(b) Gradient of the radius OP = $\frac{-6}{8} = -\frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

$y = \frac{4}{3}x + c$ through (8, −6): $-6 = \frac{4}{3} \times 8 + c$, so $c = -\frac{50}{3}$

$y = \frac{4}{3}x - \frac{50}{3}$

Answer: (a) $x^2 + y^2 = 100$ (b) $y = \frac{4}{3}x - \frac{50}{3}$