Day 49 **A Tier 10-a-day: Model Solutions**

1 Isosceles triangles and exterior angles

PQR is an isosceles triangle with PQ = PR.
QRS is a straight line.
Angle PRS = 138°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle PRQ = 180 − 138 = 42°
Base angles are equal, so angle PQR = 42°
x = 180 − 42 − 42 = 96
Answer: x = 96°

2 Map scales

The scale of a map is 1 : 25000.
Two towns are 7.5 cm apart on the map.
Work out the real distance between the towns in kilometres.

Real distance = 7.5 × 25000 = 187500 cm
187500 cm = 1875 m = 1.875 km
Answer: 1.875 km

3 Inequalities

Solve $7x - 6 \leq 2x - 11$

Collect the x terms on the left: $5x - 6 \leq -11$
Add 6: $5x \leq -5$
Divide by 5
$x \leq -1$
Answer: $x \leq -1$

4 Density

A stone has a mass of 135 g.
The density of the material is 0.5 g/cm^3.
Work out the volume of the stone.

Volume = mass ÷ density
= 135 ÷ 0.5 = 270 cm^3
Answer: 270 cm^3

5 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$$\cos x = \frac{14^2 + 8^2 - 7^2}{2 \times 14 \times 8}$$

$$\cos x = \frac{211}{224}$$

$x = \cos^{-1}(0.9420)$
$x = 19.6°$
Answer: x = 19.6°

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6 Quadratic graphs

The curve C has equation $y = -x^2 - 6x + 7$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set $y = 0$: $-x^2 - 6x + 7 = 0$, so $x^2 + 6x - 7 = 0$ (divide by -1)
$(x + 7)(x - 1) = 0$
$x = -7$ or $x = 1$, so the graph crosses the x-axis at $(-7, 0)$ and $(1, 0)$
(b) The turning point is halfway between the roots: $x = (-7 + 1) \div 2 = -3$
$y = 16$, so the maximum point is $(-3, 16)$
Answer: (a) $(-7, 0)$ and $(1, 0)$ (b) $(-3, 16)$

7 HCF and LCM

(a) Write 36 and 120 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 36 and 120.

(a) $36 = 2^2 \times 3^2$ and $120 = 2^3 \times 3 \times 5$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$
LCM: take the larger power of every prime: $2^3 \times 3^2 \times 5 = 360$
Answer: (a) $36 = 2^2 \times 3^2$, $120 = 2^3 \times 3 \times 5$ (b) HCF = 12, LCM = 360

8 Quadratic inequalities

Solve $x^2 \geq 121$

$x^2 = 121$ at $x = \pm 11$
The curve $y = x^2$ is above 121 outside the roots
$x \leq -11$ or $x \geq 11$
Answer: $x \leq -11$ or $x \geq 11$

9 Inverse proportion

12 pumps take 6 hours to empty a pond.
All the pumps work at the same rate.
How many pumps are needed to empty a pond in 24 hours?

Total work = $12 \times 6 = 72$ pump-hour
Number needed = $72 \div 24 = 3$
Answer: 3

10 Roots and powers

(a) Work out $\sqrt{3600}$
(b) Work out $\sqrt[3]{64000}$

(a) $\sqrt{3600} = \sqrt{36 \times 100} = 6 \times 10 = 60$
(b) $\sqrt[3]{64000} = 4 \times 10 = 40$
Answer: (a) 60 (b) 40