Day 44 **A Tier 10-a-day: Model Solutions**

1 Counting outcomes

A lock code uses 4 digits chosen from the digits 1 to 8.
No digit can be used more than once.
(a) How many different codes are possible?
(b) Work out the probability of guessing the code in one attempt.

(a) $8 \times 7 \times 6 \times 5 = 1680$
(b) Only 1 of the 1680 codes is correct

Answer: (a) 1680 (b) $\frac{1}{1680}$

2 Rationalising the denominator

Rationalise the denominator of $\frac{6}{2\sqrt{7}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$: $\frac{6\sqrt{7}}{2 \times 7}$

$= \frac{6\sqrt{7}}{14} = \frac{3\sqrt{7}}{7}$

Answer: $\frac{3\sqrt{7}}{7}$

3 Independent events

The probability that Lena wins a game is 0.7.
The probability that Omar wins a game is 0.8.
The two events are independent.
Work out the probability that at least one of them will win a game.

P(neither) = $0.3 \times 0.2 = 0.06$
P(at least one) = $1 - 0.06 = 0.94$
Answer: 0.94

4 Inequalities

Solve $x - 2 > 3x - 17$

Collect the x terms on the left: $-2x - 2 > -17$
Add 2: $-2x > -15$
Divide by -2. Dividing by a negative number reverses the inequality sign
$x < \frac{15}{2}$

Answer: $x < \frac{15}{2}$

5 Converting area and volume units

Change 0.02 m^3 to cm^3.

1 m = 100 cm, so 1 $m^3 = 100 \times 100 \times 100 = 1\,000\,000$ cm^3
$0.02 \times 1\,000\,000 = 20000$
Answer: 20000 cm^3

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6 Terminating and recurring decimals

Here are four fractions: $\frac{3}{8}$ $\frac{14}{16}$ $\frac{3}{10}$ $\frac{2}{7}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{3}{8}, \frac{14}{16} = \frac{7}{8}, \frac{3}{10}, \frac{2}{7}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{2}{7}$ has denominator 7, which includes a factor other than 2 and 5.

Answer: $\frac{2}{7}$

7 Straight line graphs

A straight line has equation $-3x + 2y = -15$

(a) Find the gradient of the line.

(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into y = mx + c: $2y = 3x - 15$

$y = \frac{3}{2}x - \frac{15}{2}$

The gradient is $\frac{3}{2}$

(b) On the x-axis, y = 0: $-3x = -15$, so x = 5

Answer: (a) $\frac{3}{2}$ (b) (5, 0)

8 Circle theorems: angle at the centre

X, Y and Z are points on a circle with centre O.

Angle XOY = 100°.

Work out the size of angle x.

Give a reason for your answer.

The angle at the centre is twice the angle at the circumference

x = 100 ÷ 2 = 50

Answer: x = 50°

9 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 5x - 11 = 0$ can be rearranged to give $x = \sqrt[3]{11 - 5x}$

(b) Show that the equation has a solution between x = 1 and x = 2

(a) Add 11 and subtract 5x from both sides: $x^3 = 11 - 5x$

Take the cube root of both sides: $x = \sqrt[3]{11 - 5x}$

(b) Let $f(x) = x^3 + 5x - 11$

f(1) = −5

f(2) = 7

There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 1 and x = 2

Answer: Shown: f(1) = −5, f(2) = 7 (change of sign)

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10 Vector geometry

OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
P is on OA with OP : PA = 4 : 3.
Q is on OB with OQ : QB = 4 : 3.
(a) Find $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) What do you notice about PQ and AB?

(a) $\overrightarrow{OP} = \frac{4}{7}\mathbf{a}$ and $\overrightarrow{OQ} = \frac{4}{7}\mathbf{b}$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\frac{4}{7}\mathbf{a} + \frac{4}{7}\mathbf{b} = \frac{4}{7}(\mathbf{b} - \mathbf{a})$

(b) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, so $\overrightarrow{PQ} = \frac{4}{7}\overrightarrow{AB}$

Answer: (a) $\frac{4}{7}\mathbf{b} - \frac{4}{7}\mathbf{a}$ (b) PQ is parallel to AB and $\frac{4}{7}$ times as long