Day 43 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 12 pancakes.
Flour: 480 g
Sugar: 240 g
Butter: 300 g
(a) Work out how much flour is needed for 3 pancakes.
(b) Hannah has 421 g of sugar. What is the greatest number of pancakes she can make?

(a) flour for one: 480 ÷ 12 = 40 g
For 3: 40 × 3 = 120 g
(b) sugar for one: 240 ÷ 12 = 20 g
421 ÷ 20 = 21 remainder 1, so she can make 21
Answer: (a) 120 g (b) 21

2 Loci and constructions

AB is a straight line of length 8 cm.
A point P moves so that it is always less than 3 cm from the point A.
Describe the region where P can be.

Less than 3 cm from A means inside the circle
Centre A, radius 3 cm
Answer: Inside a circle with centre A and radius 3 cm

3 Powers and roots in standard form

Work out $\sqrt{6.25 \times 10^{-4}}$
Give your answer in standard form.

$\sqrt{6.25 \times 10^{-4}} = \sqrt{6.25} \times \sqrt{10^{-4}}$
$= 2.5 \times 10^{-2}$
Answer: 2.5×10^{-2}

4 Difference of two squares

(a) Factorise $x^2 - 100$
(b) Use the same idea to work out $48^2 - 2^2$ without a calculator.

(a) Difference of two squares: $x^2 - 100 = x^2 - 10^2$
$(x - 10)(x + 10)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $48^2 - 2^2 = (48 + 2)(48 - 2)$
$= 50 \times 46 = 2300$
Answer: (a) $(x - 10)(x + 10)$ (b) 2300

5 Ratio and a difference

The ratio of boys to girls in a youth club is 9 : 10.
There are 14 more girls than boys.
Work out the total number of boys and girls.

Difference in parts = 10 − 9 = 1
1 parts = 14, so 1 part = 14 ÷ 1 = 14
Total parts = 9 + 10 = 19, so total = 19 × 14 = 266
Answer: 266

6 Midpoints and length of a line segment

P is the point (3, −5) and Q is the point (−5, −8).
(a) Find the midpoint of PQ.

(a) Midpoint = $(\frac{3 + (-5)}{2}, \frac{-5 + (-8)}{2}) = (-1, -6.5)$
Answer: (a) (−1, −6.5)

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7 Rationalising with conjugates

Rationalise the denominator of $\frac{6}{3-\sqrt{6}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 + \sqrt{6}$: $\frac{6(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})}$

Denominator: $9 - 6 = 3$

$\frac{6(3+\sqrt{6})}{3} = 2(3+\sqrt{6})$

$= 6 + 2\sqrt{6}$

Answer: $6 + 2\sqrt{6}$

8 Circles and tangents

The point P(15, −8) lies on a circle with centre O(0, 0).

(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.

(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 15^2 + (-8)^2 = 289$

(b) Gradient of the radius OP = $\frac{-8}{15}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{8}{15}) = \frac{15}{8}$

$y = \frac{15}{8}x + c$ through (15, −8): $-8 = \frac{15}{8} \times 15 + c$, so $c = -\frac{289}{8}$

$y = \frac{15}{8}x - \frac{289}{8}$

Answer: (a) $x^2 + y^2 = 289$ (b) $y = \frac{15}{8}x - \frac{289}{8}$

9 Probability with three selections

A bag contains 4 black pens and 4 blue pens.

Three pens are taken at random without replacement.

Work out the probability that all three pens are black.

P(all black) = $\frac{4}{8} \times \frac{3}{7} \times \frac{2}{6}$

$= \frac{24}{336} = \frac{1}{14}$

Answer: $\frac{1}{14}$

10 Bounds of a speed or density

A runner travels 360 m in 10.1 s. The distance is measured to the nearest 10 metres and the time to the nearest 0.1 second. (Calculator)

Work out the upper bound for the runner's speed in m/s.

Give your answer correct to 3 significant figures.

speed = distance ÷ time

For the upper bound of speed use the upper bound of distance and the lower bound of time.

365 ÷ 10.05 = 36.31841

= 36.3 m/s (3 s.f.)

Answer: 36.3 m/s