Day 41 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: isosceles triangles with radii

A, B and C are points on a circle with centre O.
Angle OAB = 43°.
(a) Work out angle AOB.
(b) Work out the size of angle x.
Give reasons for your answers.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 43°
Angle AOB = 180 − 43 − 43 = 94°
(b) The angle at the centre is twice the angle at the circumference
x = 94 ÷ 2 = 47
Answer: (a) 94° (b) x = 47°

2 Prime factors

Write 297 as a product of its prime factors. Give your answer in index form.

Divide by prime numbers in turn: 297 ÷ 3 = 99, 99 ÷ 3 = 33, 33 ÷ 3 = 11, 11 ÷ 11 = 1
297 = 3 × 3 × 3 × 11
In index form: $297 = 3^3 \times 11$
Answer: $297 = 3^3 \times 11$

3 Converting units of area and volume

A fish tank is a cuboid measuring 1.5 m by 1.2 m by 0.6 m.
Work out the capacity of the tank in litres.
($1\ m^3 = 1000$ litres)

Volume = $1.5 \times 1.2 \times 0.6 = 1.08\ m^3$
1.08 × 1000 = 1080
Answer: 1080 litres

4 Double inequalities

(a) Solve $6 < 5x - 4 < 36$
(b) Write down all the integer values of x that satisfy this inequality.

(a) Add 4 to every part: $10 < 5x < 40$
Divide every part by 5: $2 < x < 8$
(b) The integers are 3, 4, 5, 6, 7
Answer: (a) $2 < x < 8$ (b) 3, 4, 5, 6, 7

5 Tree diagrams

The tree diagram shows probabilities for two events.
(a) Work out the values of a and b.
(b) Work out the probability that Leah buys an ice cream.

(a) Probabilities on pairs of branches add up to 1.
a = 1 − 0.5 = 0.5 and b = 1 − 0.7 = 0.3
(b) P(ice cream) = 0.7 × 0.5 + 0.3 × 0.6
= 0.35 + 0.18 = 0.53
Answer: (a) a = 0.5, b = 0.3 (b) 0.53

6 Reverse compound change

The value of an investment grows by 2% each year. (Calculator)
After 2 years it is worth £36,414.
Work out its value at the start.

Yearly multiplier = 1.02
Overall multiplier = $1.02^2 = 1.0404$
Original value = £36,414 ÷ 1.0404 = £35,000
Answer: £35,000

Day 41 **A Tier 10-a-day: Model Solutions**

7 Factorising

Factorise fully $8x^2 - 200$

Take out the common factor 8: $8x^2 - 200 = 8(x^2 - 25)$

$x^2 - 25$ is a difference of two squares: $(x - 5)(x + 5)$

$8(x - 5)(x + 5)$

Answer: $8(x - 5)(x + 5)$

8 Two dice

Two fair dice are thrown.
The two scores are subtracted (smaller from larger).
Work out the probability that the difference is 4.

Outcomes with a difference that is 4: (1, 5), (2, 6), (5, 1), (6, 2)

There are 4 of these out of 36 equally likely outcomes

$\frac{4}{36} = \frac{1}{9}$

Answer: $\frac{1}{9}$

9 Circles and tangents

The point P(−6, 8) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-6)^2 + 8^2 = 100$

(b) Gradient of the radius OP = $\frac{8}{-6} = -\frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{4}{3}) = \frac{3}{4}$

$y = \frac{3}{4}x + c$ through (−6, 8): $8 = \frac{3}{4} \times (-6) + c$, so $c = \frac{25}{2}$

$y = \frac{3}{4}x + \frac{25}{2}$

Answer: (a) $x^2 + y^2 = 100$ (b) $y = \frac{3}{4}x + \frac{25}{2}$

10 Finding a rate of compound change

Ella invests £19,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 3 years the investment is worth £25,289.
Work out the annual rate of interest.

Overall multiplier = £25,289 ÷ £19,000 = 1.331

Take the cube root: $\sqrt[3]{1.331} = 1.1$

Rate = $(1.1 - 1) \times 100 = 10\%$

Answer: 10%