Day 40 **A Tier 10-a-day: Model Solutions**

1 Roots and powers

(a) Between which two consecutive whole numbers does $\sqrt[3]{82}$ lie?
(b) Work out $6^3 - 7^2$

(a) $4^3 = 64$ and $5^3 = 125$, and 82 is between these.
(b) $6^3 = 216$ and $7^2 = 49$, so $216 - 49 = 167$
Answer: (a) 4 and 5 (b) 167

2 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 2, y = 24.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 6.

(a) $y = kx^3$. Substitute x = 2 and y = 24: $24 = k \times 8$, so $k = 3$
$y = 3x^3$
(b) $y = 3 \times 6^3 = 3 \times 216 = 648$
Answer: (a) $y = 3x^3$ (b) 648

3 Ratio and a difference

The ratio of cats to dogs in an animal shelter is 4 : 5.
There are 4 more dogs than cats.
Work out the number of cats.

Difference in parts = $5 - 4 = 1$
1 parts = 4, so 1 part = $4 \div 1 = 4$
cats: $4 \times 4 = 16$
Answer: 16

4 Area of a sector

The diagram shows a sector of a circle of radius 12 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{40}{360} \times \pi \times 12^2$
$= \frac{1}{9} \times 144\,\pi$
$= 16\pi$
Answer: 16π cm^2

5 Simultaneous equations in context

3 sandwiches and 4 drinks cost £16.
5 sandwiches and 4 drinks cost £24.
Work out the cost of one drink.

Let s = cost of one sandwich in £ and d = cost of one drink in £
$3s + 4d = 16$ (1)
$5s + 4d = 24$ (2)
The d terms have the same sign, so subtract equation (1) from equation (2)
$2s = 8$, so $s = 4$
Substitute s = 4 into equation (1) to find d = 1
Answer: £1

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6 Volume of a prism

The diagram shows a prism.
The cross-section of the prism is a right-angled triangle.
Work out the volume of the prism.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times 13 \times 5 = 32.5$

Volume = area of cross-section × length
= 32.5 × 9
= 292.5

Answer: 292.5 cm^3

7 Equations with algebraic fractions

Solve $\frac{3}{(x+4)} + \frac{2}{(x-2)} = 1$

Multiply every term by $(x + 4)(x - 2)$
$3(x - 2) + 2(x + 4) = (x + 4)(x - 2)$
$5x + 2 = x^2 + 2x - 8$
$x^2 - 3x - 10 = 0$
$(x + 2)(x - 5) = 0$
$x = -2$ or $x = 5$

Answer: $x = -2$ or $x = 5$

8 Changing a ratio

A bag contains red counters and blue counters in the ratio 2 : 7.
18 more blue counters are added to the bag.
The ratio of red counters to blue counters is now 1 : 8.
How many counters were in the bag at the start?

Let red = 2x and blue = 7x. After adding: 2x : (7x + 18) = 1 : 8
$\frac{2x}{7x+18} = \frac{1}{8}$, so $8 \times 2x = 1(7x + 18)$
16x = 7x + 18, so 9x = 18, x = 2
Start: 2x + 7x = 9 × 2 = 18 counters

Answer: 18

9 Percentages of percentages

There are 200 passengers on a plane.
35% of the passengers are children.
40% of the children are under 5 years old.
How many passengers are children under 5?

Children = 35% of 200 = 70
Under 5 = 40% of 70 = 28

Answer: 28

10 Probability with three selections

A bag contains 6 red counters and 3 blue counters.
Three counters are taken at random without replacement.
Work out the probability that at least one of the three counters is blue.

P(none is blue) = P(all red) = $\frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{5}{21}$

P(at least one blue) = $1 - \frac{5}{21} = \frac{16}{21}$

Answer: $\frac{16}{21}$