Day 39 **A Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $4a^2 + 3ab - b^2$
when $a = -2$ and $b = 4$

Substitute: $4 \times (-2)^2 + 3 \times (-2) \times 4 - 1 \times 4^2$
$= 16 - 24 - 16$
$= -24$
Answer: -24

2 HCF and LCM

(a) Write 24 and 44 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 24 and 44.

(a) $24 = 2^3 \times 3$ and $44 = 2^2 \times 11$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 = 4$
LCM: take the larger power of every prime: $2^3 \times 3 \times 11 = 264$
Answer: (a) $24 = 2^3 \times 3$, $44 = 2^2 \times 11$ (b) HCF = 4, LCM = 264

3 Converting units of area and volume

A rectangular table top is 200 cm by 100 cm.
Work out its area in m^2.

Area $= 200 \times 100 = 20000$ cm^2
1 m^2 = 10 000 cm^2
$20000 \div 10\,000 = 2$
Answer: 2 m^2

4 Simultaneous equations

Solve the simultaneous equations
$6x + 2y = -26$
$4x - 3y = -13$

Multiply (1) by 3 and (2) by 2 to make the y coefficients match
$18x + 6y = -78$ (1)
$8x - 6y = -26$ (2)
The y terms have opposite signs, so add the equations
$26x = -104$, so $x = -4$
Substitute $x = -4$ into equation (1) to find $y = -1$
Answer: $x = -4$, $y = -1$

5 Distance-time graphs

The graph shows the distance travelled by a motorcyclist.
(a) For how many minutes did the motorcyclist stop?
(b) Work out the speed during the final part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 30 to 50 minutes, so the stop is $50 - 30 = 20$ minutes
(b) Final part: distance 20 km in 40 minutes

Speed = distance ÷ time = $20 \div \frac{40}{60} = 30$ km/h

Answer: (a) 20 minutes (b) 30 km/h

6 Adding and subtracting in standard form

Work out $(1.2 \times 10^2) + (5.5 \times 10^1)$
Give your answer in standard form.

Write both numbers using 10^1: $1.2 \times 10^2 = 12 \times 10^1$
$(12 + 5.5) \times 10^1 = 17.5 \times 10^1$
In standard form: 1.75×10^2
Answer: 1.75×10^2

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7 Combined means

The mean of 6 numbers is 39.
Another number is added. The mean of the 7 numbers is now 43.
Work out the number that was added.

Total of the first 6 numbers = 6 × 39 = 234
Total of the 7 numbers = 7 × 43 = 301
Number added = 301 − 234 = 67
Answer: 67

8 Standard form in context

The population of Island M is 3×10^7.
The population of Island N is 1.5×10^7.
How many times bigger is the population of Island M than the population of Island N?

Divide: $(3 \times 10^7) \div (1.5 \times 10^7)$
$= 3 \div 1.5 \times 10^0 = 2$
Answer: 2 times

9 Quadratic sequences

Here are the first five terms of a quadratic sequence.
−5, 1, 13, 31, 55
(a) Find an expression for the nth term.
(b) Work out the 20th term.

First differences: 6, 12, 18
Second differences: 6, 6, so a = 6 ÷ 2 = 3
Subtract $3n^2$ from each term (3, 12, 27, 48): −8, −11, −14, −17
This is linear: $-3n - 5$
nth term = $3n^2 - 3n - 5$
(b) Substitute n = 20: $3 \times 20^2 - 3 \times 20 - 5 = 1135$
Answer: (a) $3n^2 - 3n - 5$ (b) 1135

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 39° and angle ACD = 48°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 48°
In triangle ABE: angle BAE = 39° and angle ABE = 48°
x = 180 − 39 − 48 = 93
Answer: x = 93°