Day 358 **A Tier 10-a-day: Model Solutions**

1 Equations with brackets

Solve $5(x + 2) = -3(x + 5) + 9$

Expand the brackets: $5x + 10 = -3x - 15 + 9$
Simplify: $5x + 10 = -3x - 6$
Collect x terms on one side and numbers on the other: $8x = -16$
$x = -2$
Answer: $x = -2$

2 Sharing in a ratio

Isla and Hugo share some money in the ratio 5 : 1.
Hugo gets £12.
How much does Isla get?

1 parts = £12, so 1 part = £12 ÷ 1 = £12
Isla gets 5 parts: 5 × £12 = £60
Answer: £60

3 nth term of a linear sequence

Here are the first four terms of a sequence.
5, 8, 11, 14
(a) Find an expression for the nth term.
(b) Is 111 a term of this sequence? You must show your working.

The terms go up by 3 each time, so the nth term starts with 3n
5 − 3 = 2, so the nth term is 3n + 2
(b) 3n + 2 = 111
$3n = 109$, so $n = \frac{109}{3}$
n is not a positive whole number, so 111 is not a term of the sequence
Answer: (a) 3n + 2 (b) No

4 Circle theorems: angle at the centre

A, B and C are points on a circle with centre O.
Angle AOB = 126°.
Work out the size of angle x.
Give a reason for your answer.

The angle at the centre is twice the angle at the circumference
x = 126 ÷ 2 = 63
Answer: x = 63°

5 Income tax

No tax is paid on the first £12,500 of income. Tax of 25% is paid on the next £28,000. Tax of 35% is paid on all income above £40,500. (Calculator)
Tom earns £60,500 a year.
Work out the income tax paid.

Next £28,000 at 25%: £28,000 × 0.25 = £7,000
Above £40,500: £60,500 − £40,500 = £20,000, taxed at 35%: £20,000 × 0.35 = £7,000
Total tax = £7,000 + £7,000 = £14,000
Answer: £14,000

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6 Surface area of cones and spheres

A sphere has radius 11 cm.
Work out the surface area of the sphere.
Give your answer in terms of π.

Surface area = $4\pi r^2$
$= 4 \times \pi \times 11^2$
$= 484\pi$
Answer: 484π cm^2

7 Cumulative frequency graphs

The cumulative frequency graph shows information about the age of 80 visitors.
Use the graph to find an estimate for
(a) the median,
(b) the number of visitors with a age greater than 80 years.

(a) Median at cumulative frequency $80 \div 2 = 40$
Read across from 40 to the graph, then down to the axis
Median = 50 years
(b) At 80 years the cumulative frequency is 71
$80 - 71 = 9$
Answer: (a) 50 years (b) 9

8 Counting

5 friends stand in a line for a photograph.
(a) How many different orders are possible?
(b) How many different orders are there if the youngest must stand first?

(a) $5 \times 4 \times 3 \times 2 \times 1 = 120$
(b) The other 4 friends can be arranged in $4 \times 3 \times 2 \times 1 = 24$ ways
Answer: (a) 120 (b) 24

9 Perpendicular bisector

A is the point (8, 0) and B is the point (2, 2).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{8+2}{2}, \frac{0+2}{2}) = (5, 1)$

Gradient of AB = $\frac{2}{-6} = -\frac{1}{3}$

Perpendicular gradient = $-1 \div (-\frac{1}{3}) = 3$

$y = 3x + c$ through (5, 1): $1 = 3 \times 5 + c$, so $c = -14$
$y = 3x - 14$
Answer: $y = 3x - 14$

10 Bearings with the cosine rule

Boat P sails 7 km from a harbour on a bearing of 085°.
Boat Q sails 25 km from the same harbour on a bearing of 195°. (Calculator)
Work out the distance between the two boats.
Give your answer to 3 significant figures.

The angle between the two paths = $|85° - 195°| = 110°$
$d^2 = 7^2 + 25^2 - 2 \times 7 \times 25 \times \cos 110°$
$d^2 = 793.71$
$d = 28.2$
Answer: 28.2 km