Day 356 **A Tier 10-a-day: Model Solutions**

1 Two-way tables

The table shows information about 71 people.
One person is chosen at random.
(a) Find the probability that the person is a child and chose coffee.
(b) The person chose tea. Find the probability that they are an adult.

(a) Children who chose coffee: 20 out of 71 = $\frac{20}{71}$

(b) Tea total = 33, of whom 9 are in the Adults row: $\frac{3}{11}$

Answer: (a) $\frac{20}{71}$ (b) $\frac{3}{11}$

2 Surface area of cones and spheres

A solid hemisphere has radius 2 cm.
Work out the total surface area of the hemisphere.
Give your answer in terms of π.

Curved surface = $\frac{1}{2} \times 4\pi r^2 = 8\pi$

Flat circular face = $\pi r^2 = 4\pi$
Total = 12π
Answer: 12π cm^2

3 Negative and fractional indices

Work out the value of

(a) $(\frac{1}{2})^{-2}$

(b) $1000^{\frac{-2}{3}}$

(a) $(\frac{1}{2})^{-2} = 2^2 = 4$

(b) $1000^{\frac{-2}{3}} = \frac{1}{1000^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{1000})^2} = \frac{1}{10^2} = \frac{1}{100}$

Answer: (a) 4 (b) $\frac{1}{100}$

4 Forming and solving equations

Priya is x years old.
Her aunt is 4 times as old as Priya, plus 1 years.
The sum of their ages is 56 years.
Work out how old her aunt is.

Let Priya's age be x, so her aunt's age is 4x + 1
x + (4x + 1) = 56
5x = 55
x = 11
Her aunt's age = 4 × 11 + 1 = 45
Answer: 45 years

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5 Area of a sector

The diagram shows a sector of a circle of radius 7 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{270}{360} \times \pi \times 7^2$

= $\frac{3}{4} \times 49\ \pi$

= $\frac{147}{4}\pi$

Answer: $\mathbf{\frac{147}{4}\pi\ cm^2}$

6 Finding a missing value from an average

The mean of the five expressions below is 9.
3x, x + 6, 7, 4x, 8
Work out the value of x.

Total = 5 × 9 = 45
Add the expressions: 8x + 21 = 45
8x = 24, so x = 3
Answer: x = 3

7 Adding algebraic fractions

Write as a single fraction

$\frac{-2}{(x-5)} - \frac{1}{(x+4)}$

Common denominator is $(x - 5)(x + 4)$

= $\frac{-2(x+4) - 1(x-5)}{(x-5)(x+4)}$

= $\frac{-3x - 3}{(x-5)(x+4)}$

Answer: $\mathbf{\frac{-3x - 3}{(x-5)(x+4)}}$

8 Standard form in context

One rice grain has a mass of 2×10^{-6} g.
A sample contains 3×10^8 rice grains.
Work out the total mass of the sample.
Give your answer in standard form.

Total mass = $(2 \times 10^{-6}) \times (3 \times 10^8)$
$2 \times 3 = 6$ and $10^{-6} \times 10^8 = 10^2$
Answer: $\mathbf{6 \times 10^2}$ g

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9 Circles and tangents

The point P(−6, −8) lies on a circle with centre O(0, 0).

(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.

(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-6)^2 + (-8)^2 = 100$

(b) Gradient of the radius OP = $\frac{-8}{-6} = \frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{4}{3} = -\frac{3}{4}$

$y = -\frac{3}{4}x + c$ through (−6, −8): $-8 = -\frac{3}{4} \times (-6) + c$, so $c = -\frac{25}{2}$

$y = -\frac{3}{4}x - \frac{25}{2}$

Answer: (a) $x^2 + y^2 = 100$ (b) $y = -\frac{3}{4}x - \frac{25}{2}$

10 Bounds and accuracy

A cyclist travels 720 metres in 18.9 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.

(a) Work out the upper bound and the lower bound of the speed in m/s.

(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 725 ÷ 18.85 = 38.4615
Lower bound = 715 ÷ 18.95 = 37.7309

(b) Both bounds round to 38 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 38 m/s to 2 significant figures.

Answer: (a) 38.4615, 37.7309 (b) 38 m/s (2 s.f.)