Day 355 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 63°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 63 = 27
Answer: x = 27°

2 Percentage change

The number of members of a club changes from 400 to 640.
Work out the percentage increase.

Increase = 640 − 400 = 240
Percentage increase = $\frac{240}{400} \times 100 = 60\%$
Answer: 60%

3 Estimation

By rounding each number to 1 significant figure, estimate the value of
$\frac{72.0 \times 30}{0.48}$

Round each number to 1 significant figure: 70, 30, 0.5
$\frac{70 \times 30}{0.5} = 4200$
Answer: 4200

4 Is it a right-angled triangle?

A triangle has sides of length 10 m, 26 m and 24 m.
Is the triangle right-angled?
You must show your working.

Longest side^2 = 26^2 = 676
Sum of the squares of the other two sides = $10^2 + 24^2 = 100 + 576 = 676$
676 = 676, so the triangle is right-angled
Answer: Yes, it is right-angled

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
9, 19, 35, 57, 85
(a) Find an expression for the nth term.
(b) Work out the 8th term.

First differences: 10, 16, 22
Second differences: 6, 6, so a = 6 ÷ 2 = 3
Subtract $3n^2$ from each term (3, 12, 27, 48): 6, 7, 8, 9
This is linear: n + 5
nth term = $3n^2 + n + 5$
(b) Substitute n = 8: $3 \times 8^2 + 1 \times 8 + 5 = 205$
Answer: (a) $3n^2 + n + 5$ (b) 205

6 HCF and LCM in index form

A = $2^2 \times 5^2 \times 7^2$

B = $5^2 \times 7^4$

Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.

Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $5^2 \times 7^2$

LCM: for each prime take the higher power: $2^2 \times 5^2 \times 7^4$

Answer: HCF = $5^2 \times 7^2$, LCM = $2^2 \times 5^2 \times 7^4$

7 Simple and compound interest

Zac and Owen each invest £2500 for 2 years. (Calculator)

Zac's account pays 2% simple interest per year.

Owen's account pays 2% compound interest per year.

Work out how much more interest Owen earns than Zac.

Simple interest = £2500 × 2% × 2 = £100

Compound: £2500 × 1.02^2 = £2601.00, so interest = £101.00

Difference = 101.00 − 100 = £1.00 (to the nearest penny)

Answer: £1.00

8 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.

Find the value of b and the value of c.

The curve crosses the x-axis at x = −2 and x = −1

So $y = -(x + 2)(x + 1)$

Expand: $y = -x^2 - 3x - 2$

b = −3 and c = −2

Answer: b = −3, c = −2

9 Equations with algebraic fractions

Solve $\frac{3}{(x + 5)} + \frac{4}{(x - 5)} = 1$

Multiply every term by $(x + 5)(x - 5)$

$3(x - 5) + 4(x + 5) = (x + 5)(x - 5)$

$7x + 5 = x^2 - 25$

$x^2 - 7x - 30 = 0$

$(x + 3)(x - 10) = 0$

x = −3 or x = 10

Answer: x = −3 or x = 10

10 Inverse proportion formulae

y is inversely proportional to x.

When x = 25, y = 10.

(a) Find a formula for y in terms of x.

(b) Work out the positive value of x when y = 25.

(a) $y = \frac{k}{x}$. Substitute: $10 = \frac{k}{25}$, so k = 10 × 25 = 250

$y = \frac{250}{x}$

(b) $25 = \frac{250}{x}$, so x = 250 ÷ 25 = 10

Answer: (a) $y = \frac{250}{x}$ (b) x = 10