Day 351 **A Tier 10-a-day: Model Solutions**

1 Functions

f(x) = 2x + 6
g(x) = x^2 + 1
Find fg(−6)

g(−6) = $(-6)^2$ + 1 = 37
f(37) = 2 × 37 + 6 = 80
Answer: 80

2 Primes

(a) Write down all the prime numbers between 90 and 108.
(b) Write 209 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 90 and 108 gives: 97, 101, 103, 107
(b) 209 ÷ 11 = 19, and both 11 and 19 are prime.
Answer: (a) 97, 101, 103, 107 (b) 11 × 19

3 Rearranging formulae

Make m the subject of n = 2(m + 3)

Divide both sides by 2: $\frac{n}{2}$ = m + 3

Subtract 3 from both sides

m = $\frac{n}{2}$ − 3

Answer: m = $\frac{n}{2}$ − 3

4 Trigonometry: finding a side

Work out the value of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

x is the hypotenuse and 12 m is the opposite, so use sin

sin 31° = $\frac{12}{x}$

x = 12 ÷ sin 31°
x = 23.3
Answer: x = 23.3 m

5 HCF and LCM in index form

A = $2^4 \times 3^3 \times 5$
B = $2^4 \times 3^3$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2^4 \times 3^3$
LCM: for each prime take the higher power: $2^4 \times 3^3 \times 5$
Answer: HCF = $2^4 \times 3^3$, LCM = $2^4 \times 3^3 \times 5$

6 Angles in regular polygons around a point

A regular pentagon and an equilateral triangle share the side PQ and are on opposite sides of it.
The two shapes meet at the point P.
Work out the size of the angle at P that is not inside either shape.

Interior angle of the regular pentagon = 180 − 360 ÷ 5 = 108°
Interior angle of the equilateral triangle = 180 − 360 ÷ 3 = 60°
Angles round a point add up to 360°
360 − 108 − 60 = 192
Answer: 192°

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7 Surd fractions

Show that $\frac{\sqrt{125}-\sqrt{80}}{\sqrt{5}}$ is an integer, and find its value.

$\sqrt{125} = 5\sqrt{5}$ and $\sqrt{80} = 4\sqrt{5}$

Numerator $= 5\sqrt{5} - 4\sqrt{5} = \sqrt{5}$

$\frac{\sqrt{5}}{\sqrt{5}} = 1$

Answer: 1

8 Reading a histogram

The histogram shows information about the time of some items.

(a) Work out the number of items with a time in the class 50 to 60 minutes.

(b) Work out an estimate for the number with a time greater than 25 minutes.

(a) Frequency = frequency density × class width

$2.2 \times 10 = 22$

(b) 25 to 40: $1.4 \times 15 = 21$

Class 40 to 50: $2 \times 10 = 20$

Class 50 to 60: $2.2 \times 10 = 22$

Total $= 21 + 20 + 22 = 63$

Answer: (a) 22 (b) 63

9 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$y = x^2 + 2x + 10$

$y = -2x + 7$

Set the y values equal: $x^2 + 2x + 10 = -2x + 7$

$x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x = -3$ or $x = -1$

Substitute into $y = -2x + 7$: $y = 13$ and $y = 9$

Answer: $x = -3$, $y = 13$ and $x = -1$, $y = 9$

10 Probability with three selections

A bag contains 4 black pens and 3 blue pens.

Three pens are taken at random without replacement.

Work out the probability that at least one of the three pens is blue.

P(none is blue) = P(all black) $= \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} = \frac{4}{35}$

P(at least one blue) $= 1 - \frac{4}{35} = \frac{31}{35}$

Answer: $\frac{31}{35}$